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Extreme rainfall analysis and estimation of Probable Maximum Precipitation (PMP) by statistical methods over the Indus river basin in India

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Abstract Annual extreme rainfall series of 1-3 day durations at stations located inside the Indus basin in India were subjected to statistical analysis in order to estimate point Probable Maximum Precipitation (PMP) and maximum rainfall of different return periods for the durations of 1-3 days. Daily rainfall data of 210 stations ranging from 1901-2000 (with varying data length) has been considered for the present study. Rainfall distribution of the basin on seasonal and annual scale has been examined. Spatial patterns of 1-3 day extreme rainfall over the basin have been presented which showed that low values of extreme rainfall are located in the Ladakh region located in the northern parts of the basin, while region lying in the Himachal Pradesh (Sutlej river basin) experienced heavy rainfall. Over the entire basin, point PMP estimates were found to range from about 5 to 98 cm for 1-day, 7 to 137 cm for 2-day and 8 cm to 163 cm in 3-day durations. Highest values of point PMP for 1-3 day durations were found to correspond to Kilba station in the Sutlej basin. Extreme Value Type-I (EV1) distribution has been fitted to 1-3 day extreme rainfall series and various return period values were estimated. Using the same fit it was found that, PMP estimates for 1-3 day durations, have return period of the order of 1000-year. Extreme rainfall features and estimates of point PMP and maximum rainfall for different return periods documented in this study will be useful for designing and planning the water resources projects in the basin.

Key Words: Seasonal Rainfall**,** Extreme Rainfall, Extreme Value Type–I Distribution, Return Period, Probable Maximum Precipitation.

1. Introduction

The need for flood control in the foothill areas of the Himalayan region to protect low land interests is the primary motivation for large-scale water resources projects in Himalayan river basins like the Indus basin. However, with the transformation of agricultural and industrial economics of the plains, demands for irrigation and hydroelectricity have accelerated. Dam construction has been the most intensive on the Himalayan rivers like the Ganges and the Indus river systems for a variety of reasons, including proximity to urban centers, fertile agricultural lands and availability of water storage sites.

Dams, barrages and other hydraulic structures across the rivers must be designed and built to withstand maximum floods that can occur at a site. Proper management of the water resources for various purposes such as for domestic use, industrial use, irrigation, hydropower generation are of prime importance. PMP is the key design rainfall input in computing Probable Maximum Floods (PMF). If a spillway is not able to safely release the PMF, breaching of the dam wall due to overtopping can occur and cause heavy loss of lives and damages to property. Also for planning and management for hydraulic structures of medium and

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minor nature such as bridges, culverts, storms drainage works etc., require estimates of design rainfall of specific return periods.

Considering the importance of the subject, an attempt has been made in this study to carry out extreme rainfall analysis of 210 stations in the Indus basin in India by statistical methods to estimate point PMP and maximum rainfall of different return periods which are often needed for proper planning, management and design of different types of water resources projects in a river basin.

2. Physiographic features of the Indus basin

The Indus river system comprises of the main river Indus and its major tributaries, the Kabul, the Swat and the Kurd from the west and the Jhelum, the Chenab, the Ravi, the Beas and the Sutlej from the east. The main river of the system, the mighty Indus rises in Tibet near Mansarovar Lake at an elevation of 5180 m, passes through inaccessible mountain ranges in northern Kashmir and Gilgit, enters Pakistan and emerges out of the hills near Attock (Rao,1975). After flowing for a distance of about 2880 km it meets the Arabian Sea. The entire Indus basin extends over an area of about 11,54,500 km^2 out of which the drainage area lying in India is about 3,21,290 km^2 (nearly 9.8 % of the total geographical area). The basin lies in the states of Jammu & Kashmir (approx. area 1,93,762 km^2), Himachal Pradesh (51,356 km^2), Punjab (50,304 km^2), Rajasthan (15,814 km^2), Haryana (9,939km^2 .) and Union Territory of Chandigarh (114 km^2.). The upper reaches of the Indus river are remote and relatively inaccessible. The entire Indus basin has been divided into seven sub-basins 201-207 (Khosla, 1949). Details of these sub-basins are given in Table – 1

Table 1: Description of Sub-basins of Indus river, area (in km^2) and number of rainfall stations.

Sub-basin	Description	Area in km^2	No. of stations
201	River Sutlej upto Bhakra Dam Site	23044	18
202	River Sutlej between Bhakra Dam site and Beas excluding Beas	67398	84
203	River Beas	20894	28
204	River Ravi	14834	2
205	River Chenab	29493	30
206	River Jhelum	29901	35
207	River Indus as far as Pakistan boundary	135726	13
	Total	321290	210

3. Data used and Methodology

Daily rainfall data of 210 stations for the period of 1901-2000 (with varying data length) located inside the Indus basin in India were considered for this study. Location of these rainfall stations and demarcation of different sub-basins are shown in Fig. 1 along with the topographical features of the basin. Description of sub-basins, corresponding area and data availability of these sub-basins are given in Table-1. Rainfall features such as seasonal rainfall, annual rainfall, highest rainfall amounts, PMP and maximum rainfall of different return periods for 1-3 day durations were estimated for each of 210 stations considered in the study. These estimates were then interpolated at regular grids at the resolution of $0.1^{0}\ {}^{x}\ 0.1^{0}$ using the inverse squared distance method. Spatial patterns of these features were displayed on the basin map using GrADS software (Version 1.8)

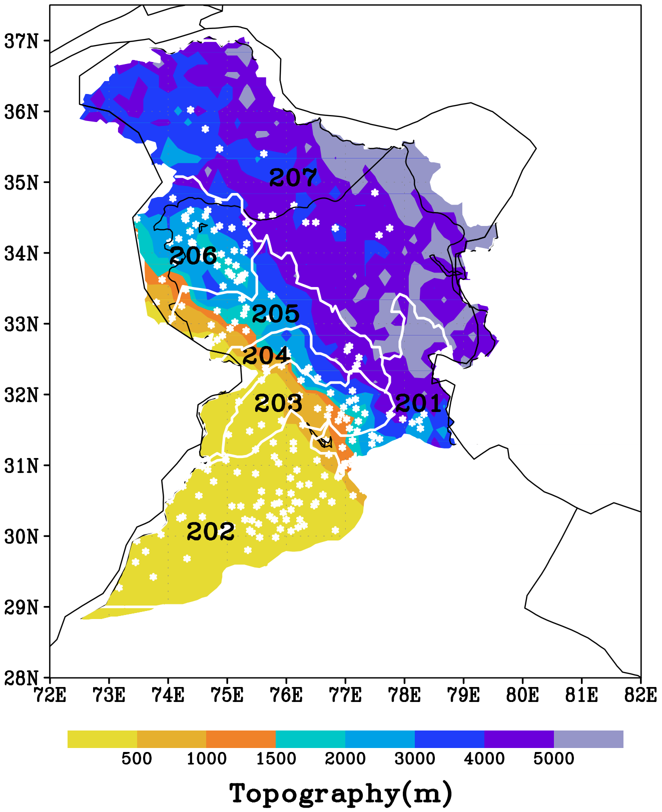


Fig.1: Location of rainfall stations over different sub-basins superimposed over topography of the Indus basin

3.1 GrADS Software Version 1.8

GrADS (Grid Analysis and Displaying System) is a software used to display the results on the map of a place. It is an interactive desktop tool that is used for easy access and visualization of earth science data. The format of data may be either binary, GRIB, NetCDF or HDF-SDS (Scientific Data Sets) has been implemented worldwide on a variety of commonly used operating systems and is freely distributed over the internet. GrADS uses a 4-dimensional data environment: longitude, latitude, vertical level and time. GrADS interprets station data as well as gridded data and the grids may be regular, non-linearly spaced, Gaussian or of variable resolution. Operations are executed interactively by entering FORTRAN like expressions at the command line. Data may be displayed using a variety of graphical techniques: line and bar graphs, scatter plots, smoothed contours, shaded contours, streamlines, wind vectors, grid boxes, shaded grid boxes and station model plots. (website : www.iges.org/grads)

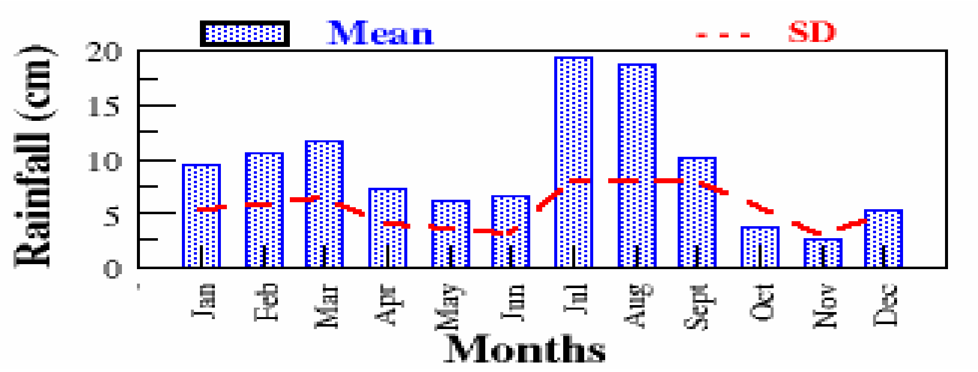
4. Seasonal rainfall distribution over the Indus basin

Rainfall in the Indus basin varies over different sub-basins. It also varies with elevation. The Himalayas act as a major barrier for the natural flow of the southwest monsoon. Hence the monsoon takes an entirely different course and the dominance of monsoon is not uniform over different portions of the basin. At meso-scale, the rainfall is highly influenced by local orography. Rainfall, in general increases from low land valleys to higher mountain slopes upto certain heights. The windward slope gets more rainfall than the leeward side.

Considering different rain-bearing systems in the basin, seasons may be broadly classified as (i) Winter-December to March, (ii) Summer/Pre-monsoon-April to June, (iii) Southwest Monsoon - July to September and (iv) Post-Monsoon/ Autumn- October to November. Northern portion of the basin receives rainfall in the winter season due to Western Disturbances while southern portion dominated by south-west monsoon circulations. Fig. 2 gives the annual cycle of the rainfall over the Indus river basin. It is clearly seen from this figure that, rainfall occurs in all the months of the year, but winter months and south-west monsoon months receive substantial amount of rainfall. Southwest monsoon starts in the month of July and withdraws from the basin by about middle of September. Mean seasonal and annual rainfall and their contribution to annual rainfall over different sub-basins and the entire Indus basin have been estimated and are given in table-2. It is seen from table-2 that mean rainfall during July to September over southern portion of the Indus river basin ranges from about 45 cm in sub-basin 202 to 102 cm in sub-basin 203 while northern parts of the basin (i.e. sub-basins 206 and 207) receive comparatively less amount of rainfall during monsoon season (18.4 cm and 2.7 cm respectively). Mean annual rainfall over sub-basin 203 was found to be the highest (160 cm) and the lowest over sub-basin 207 (19 cm). However, considering mean annual rainfall at individual station it was seen that the highest mean annual rainfall of the order of 310 cm was observed at Dharamsala located in the hilly region in sub-basin 203 and lowest annual rainfall of 5cm was observed at Khalatse station in sub-basin 207. Spatial patterns of two important season over the study region viz., Southwest monsoon season (July-September), and Winter season (December-March) along with the annual rainfall are shown in Fig.3 (a, b, c). From this figure it is seen that north-eastern part (Ladakh region) of the basin is devoid of rainfall due to unavailability of moisture to precipitate over the region (annual rainfall is 19cm). In the winter season, central part of the basin (sub-basins 205 and 206) experiences good rainfall activity due to the passage of Western Disturbances. It is of the order of 39cm to 42 cm (31-40% of the annual rainfall total at that location).

5 Meteorological causes of heavy rainfall over the basin

The basin under study is characterized by different climatic conditions from tropical to alpine. The upper portion of the basin (north-western part) receives good rainfall during the winter season due to the passage of Western Disturbances across the Himalayas. These disturbances are eastward moving low-pressure areas or upper air troughs in the subtropical westerlies (Pisharoty and Desai, 1956). During winter, the frequency of these disturbances is of the order of 4 to 6 per months and this frequency reduces as the season advances (Dhar et. al, 1987). With the setting in of winter season, these disturbances have a tendency to move along lower latitudes and with the advance of the season they tend to move along the higher latitudes. As such bulk of precipitation occurs in the Western Himalayas that is in Kashmir valley, Himachal Pradesh, Punjab, West U.P. Hills and adjoining plains. The precipitation associated with these disturbances decreases sharply as they move from west to east along the Himalayas. The summer or pre-monsoon season lasts for about 3 months from April to June. This season is the transitional period before the onset of southwest monsoon. Western Disturbances do occur in this season but their average frequency is about 2--3 per month (Dhar et. al, 1987).



. Fig. 2 : Annual cycle of the rainfall over the Indus river basin

Table-2: Mean seasonal and annual rainfall (with % contribution to annual) over different sub-basins and entire Indus basin

Sub-basin	***Seasonal Rainfall***								
	Winter (Dec.- Mar.)		Pre-Monsoon (Apr-Jun)		Monsoon (July-Sept.)		Post-Monsoon (Oct. –Dec.)		Annual
	Rainfall (cm)	% of annual	Rainfall (cm)	% of annual	Rainfall (cm)	% of. annual	Rainfall (cm)	% of annual	Rainfall (cm)
201	30.3	25	22.3	19	62.2	52	5.3	4	120
202	8.4	13	7.7	12	44.5	71	2.5	4	63.1
203	29.2	18	23.5	15	101.7	63	5.8	4	160.2
204	29.3	20	18.6	12	94.5	64	6.8	4	149.2
205	41.8	31	23.9	17	65.2	48	5.2	4	136.2
206	39.2	40	27.4	28	18.4	19	12.5	13	97.6
207	9.0	47	5.6	29	2.7	14	2.1	11	19.3
Entire	22.4	24	16.3	18	50.1	54	4.2	5	93.0

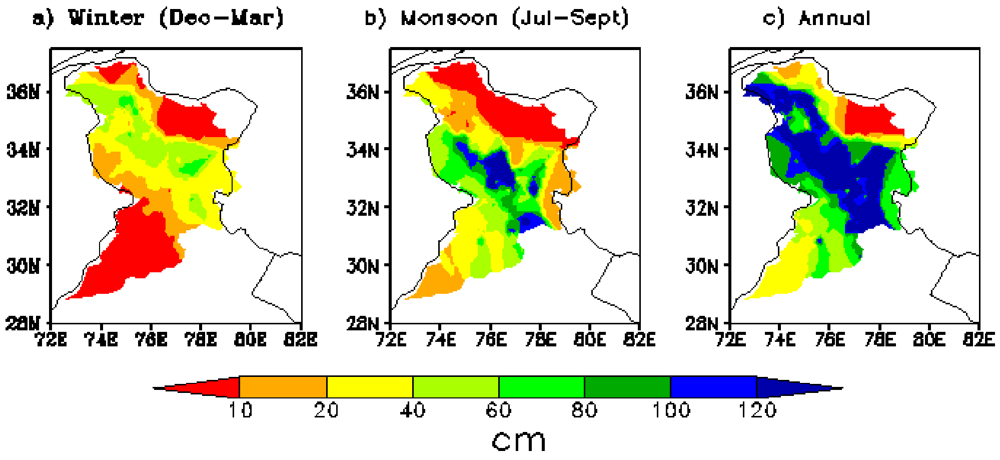


Fig. 3 : Spatial patterns of mean seasonal (Winter and Monsoon) and Annual rainfall (cm) over the Indus basin.

During the southwest monsoon months of July to September, sub-basins 201 to 205 come under the sway of moist monsoon current from the Bay of Bengal/the Arabian Sea. The area in sub-basin 206 (Jhelum) falls in Kashmir valley has saucer shaped with steep mountain slopes all round, therefore, heavy rainfall even for durations of 1 to 2 days can cause serious floods. For the sub-basin 207 falling in the Ladakh region is located in the worst arid region of India due to lack of precipitation. Precipitation is received in all the months but with very negligible quantity. As such no specific season may be mentioned in this region.

During the period July to September moderate to heavy rainfall occurs over the Indus basin in association with the following weather situations:

(a) Re-curving monsoon depressions or low pressure areas from the head Bay of Bengal or the Arabian Sea dissipating over the basin,

(b) Movement of westerly waves (or Western Disturbances) over the northern portion of the basin synchronizing with the passage of monsoon disturbances in the lower latitudes,

(c) Movement of upper air cyclonic circulations over the basin and or

(d) Accentuation of the western end of the seasonal trough over and near the foothills of the Himalayan region during break monsoon situations.

6. Highest Observed Point Rainfall

Magnitudes of highest rainfall over a region or a river basin are useful as background material for researchers in the estimation of PMP or extreme rainfall that is possible at a given point for a given duration. There are many studies available on highest rainfall over different parts of India. However detailed catalogue of highest rainfall over the study river basin is lacking. The map showing the distribution of one-day highest point rainfall in India was first prepared by Iyer and Zafer (1938) based on the data of 30 years from 1891 to 1920. Several authors have subsequently carried out studies on highest rainfall over Indian region. Rakhecha et al. (1990) have prepared homogeneous zones of highest rainfall for 1-day duration over India by considering 300 stations data during 1875 to 1982. Mandal et.al (2006) updated the highest 1,2 & 3-day rainfall over Indian region considering rainfall data of 643 stations in India during 1901 to 1991.

Due to the interest of highest rainfall distribution in different durations and their usefulness in many types of hydrologic analyses over a river basin, spatial patterns of highest ever-recorded 1, 2 and 3-day rainfalls for the Indus basin have been prepared and the same are shown in Fig. 4 (a, b, & c). The extreme rainfall values were found to range from 2 to 61 cm for 1-day duration, from 3 to 105 cm in 2-day from 4 to 106 cm in 3-day durations. Range of highest observed rainfall values and name of the highest rainfall recording station corresponding to durations of 1 to 3 days in each sub-basin and the entire Indus basin is given in Table– 3.

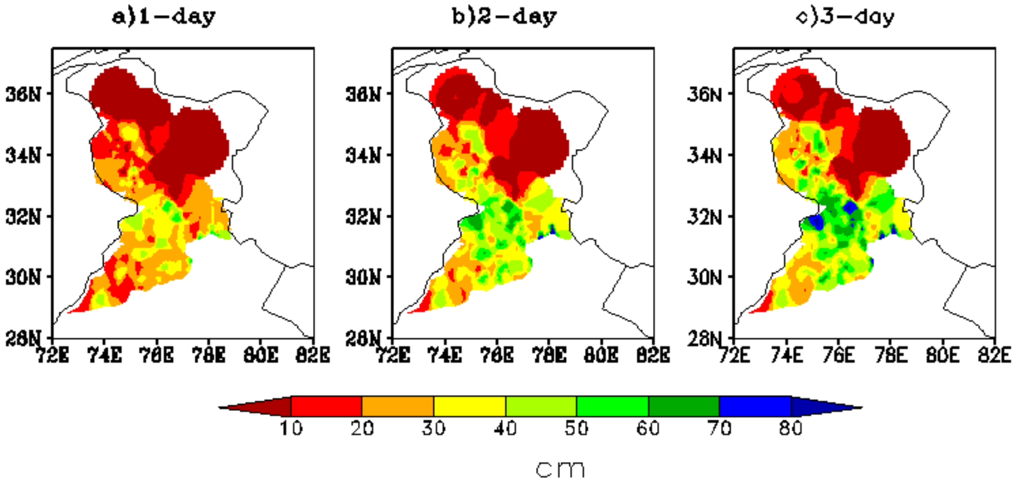


Fig.4 : Spatial patterns of extreme rainfall (cm) for 1,2 & 3-day durations over the Indus basin

Table–3: Ranges of highest observed rainfall (cm) in different sub-basins. Station which has recorded highest value has been shown in the bracket.

Sub-basin	**Range of Highest observed rainfall (cm)**		
	1-day	**2-day**	**3-day**
201	14-61 (Kilba)	27-105 (Kilba)	29-106 (Kilba)
202	13-56 (Nahan)	14-65 (Nahan)	16-80 (Nahan)
203	13-60 (Dharmsala)	21-70 (Hamirpur)	25-80 (Dharmsala)
204	23-41 (Chamba)	34-56 (Chamba)	35-68 (Chamba)
205	10-37 (Banihal)	13-63 (Banihal)	21-74 (Banihal)
206	15-43 (Quazigund)	19-50 (Quazigund)	12-53 (Quazigund)
207	2-11 (Dras)	3-20 (Dras)	3-30 (Dras)

7. Estimation of PMP by Statistical Method

Statistical method used for estimating point PMP at a station or over an area is based upon the assumption that information regarding extreme rainfall is contained in the long rainfall records of that station/area. PMP is defined as theoretically the greatest depth of precipitation for a given duration that is physically possible for a given size storm area at a particular geographical location at a certain time of the year (World Meteorological Organization, 1986). Another definition of PMP which is of operational nature, states that PMP is that magnitude of rainfall which will yield flood flows of which there is virtually no risk of being exceeded (Myers, 1967).

Hershfield (1961) for the first time used the statistical method for the estimation of PMP for USA. Preliminary appraisal of this technique in Canada (Bruce and Clark, 1966) and in USA (Myers, 1967), has shown that the PMP estimates obtained by this approach are closely comparable to those obtained by the conventional moisture maximization and storm transposition methods. According to Wiesner (1970) this method has an advantage of taking into account the entire rainfall dataset, expressing it in terms of statistical parameters. The World Meteorological Organization (WMO) in their various manuals and technical publications (World Meteorological Organization, 1969, 1970, 1986) have also recommended this method for estimation of point PMP for those river basins whose daily rainfall data are available for a long-period of time. Koutsoyiannis (1999) studied the probabilistic view of Hershfield's method for estimating probable Maximum Precipitation. Papalexiou and Koutsoyiannis (2006) studied the maximum precipitation depths derived by moisture maximization method at few stations in Netherlands, they concluded that probabilistic approach for the estimation of extreme precipitation value is more consistent to the natural behavior and provides better grounds for the estimation.

This study uses the same Hershfield technique for estimating point PMP at individual stations over the Indus river basin.

A brief description of Hershfield technique for estimating point PMP for a station is as follows;

$X_{PMP} = \square X_n + S_n . K_m$ -------------(1)

where, X_{PMP} = PMP estimate for a station, $\square X_n$ = mean of the annual extreme series, S_n = standard deviation of the annual extreme series, and K_m= frequency factor which depends upon the availability of data period and return period.
Frequency factor 'K_m' is obtained by using the equation :

$K_m = (X_{max} - \square X_{n-1}) / S_{n-1}$ ----------(2)

where, X_{max} = largest value of the annual extreme series, $\square X_{n-1}$ = mean of the annual maximum series omitting the largest value from the series, and S_{n-1}= standard deviation of the annual extreme series omitting the largest value from the series. K_m values for all stations are then plotted against the $\square X_n$ values respectively and a smooth envelope curve is drawn. $K_{envelope}$ value is picked up from the curve for each station's $\square X_{n.}$ PMP for each station is then calculated using equation (1)by replacing K_m with $K_{envelope}$ value. Details of the Hershfield technique are available in various publications (World Meteorological Organization (WMO), 1986, Dhar el al. 1975).

Here K_mVs□X_n were plotted separately for each sub-basin. PMP estimates thus obtained by applying the Hershfield technique for each of 210 stations for 1, 2 and 3-day durations are then plotted over base map of the study basin and generalized PMP maps have been prepared. Fig.5 (a, b & c) shows the generalized point PMP maps by statistical method for durations of 1, 2 & 3-day over the basin. Over the entire basin point PMP estimates were found to range from about 5 to 98 cm for 1-day, 7 to 137 cm for 2-day and 8 to 163 cm in 3-day durations.

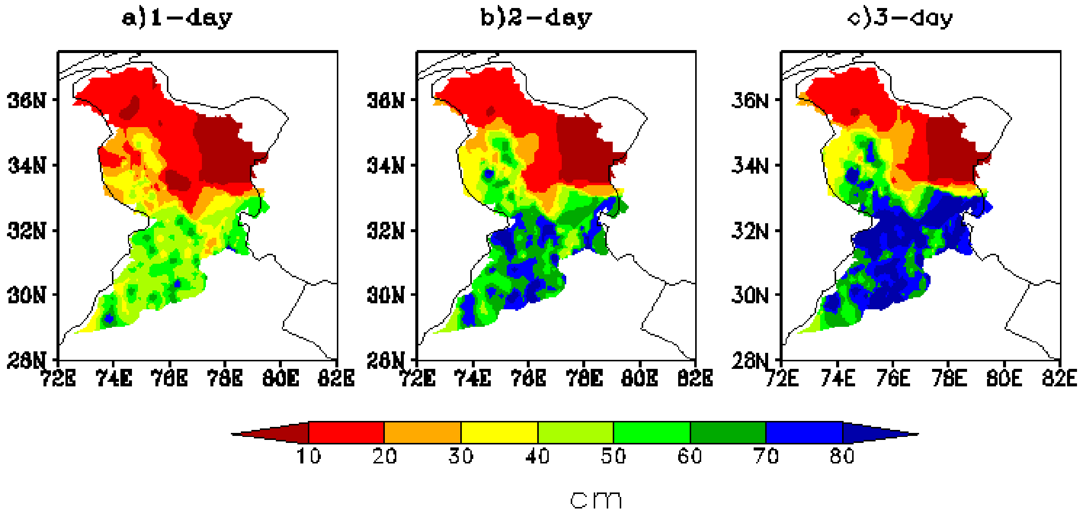


Fig.5 : Spatial patterns of 1,2 & 3-day PMP (cm) by statistical method over the Indus basin

According to Hershfield (1962) the magnitude of point PMP at an individual station should normally not exceed three times the highest observed rainfall from a long period of rainfall data and the same has also been observed from the present study. Dhar et al (1981) noted that at some of the stations over India, the highest ever-recorded 1-day recorded rainfall values were nearly equal to their respective estimates of point PMP. As an example, Bhagamandala station (Karnataka State in peninsular India) recorded the highest 1-day rainfall of 84 cm on 25 July, 1924 and it was found that its estimate of point PMP was near about this magnitude. Similarly, Bhir, a station in Maharashtra state (also in peninsular India) has recorded 35 cm rainfall on 24 July, 1989 which was found to be almost equal to the estimate of physical upper limit of rainfall (i.e. PMP) for that station (Kulkarni et al,1991).

8. Fitting of Gumbel Distribution (Extreme Value Type–I) for the estimation of maximum rainfall of different return periods

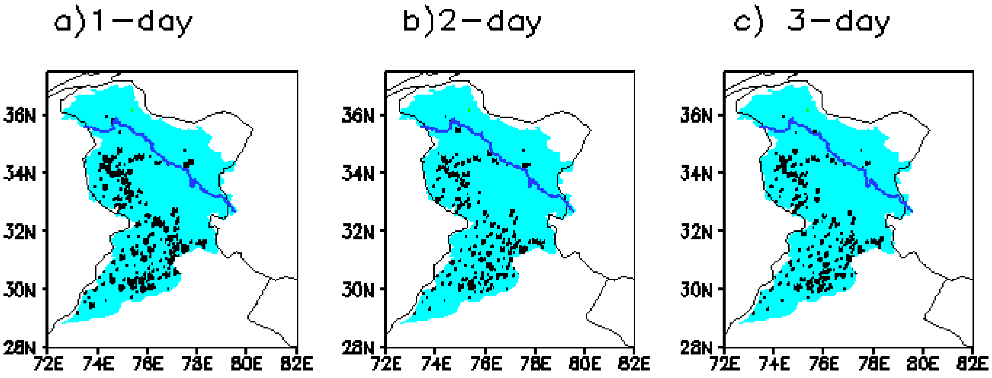
Important factors to be taken into consideration in the designing of engineering structures are safety, economy and efficiency. Thus structures have to be designed to withstand the pressures to which they may be subjected during their estimated economic life, which may vary from less than 10 years to more than 1000 years or so. Thus, design of water control structures focuses on the prediction of runoff during intense rainfall events. This requires estimates of the frequencies of occurrence of rainfall of a given duration and intensity, for analysis of the potential costs and benefits of building adequate controls. These estimates are called return periods values. The Recurrence Interval or Return Period (T) is the average time interval between the successive occurences or exceedences of an event. It is nothing but the reciprocal of the probability of exceedence of that event. The standard method of estimating the probability of occurrence of maximum rainfall for hydrologic design is to fit a known probability distribution to the maximum annual rainfall series and estimating its parameters by statistical procedures. Various theoretical and empirical distributions have been proposed by various hydrologists, which are generally applicable to the annual maximum rainfall series. A comprehensive study of various distributions was made by Hershfield and Kohlar (1960) who found that the Extreme Value Type-I (EV1) or Gumbel distribution (Gumbel, 1954) is the most suitable one. The same has been used in this study.

8.1 Extreme Value Type- I (EV1) distribution or Gumbel distribution

If X follows Extreme value Type –I or Gumbel distribution then its Cumulative Distribution Function is given by,

$F(x) = \exp\{ - \exp[- (x - u) / \alpha] \}$----------(3) , $-\infty < x < \infty$, $-\infty < u < \infty$ and $\alpha > 0$.

Here 'u' and 'α' are the location and the scale parameters of the distribution which are estimated from the observed annual extreme rainfall series using the method of least squares. Kolmogorov-Smirnov test is used for testing the Gumbel fit for 1-3 day extreme rainfall series at individual station. Fig.6 (a, b, c) shows the goodness of fit testing. It has been observed that nearly 80% stations follow Gumbel fit at 5% level of significance. Therefore further it is used for the estimation of return period values.



(. Represents good fit and • represents bad fit at 5 % level)

Fig.6 : Gumbel fit for 1-3 day duration extreme rainfall series at each station

8.2 Estimation of return period values

Inverting the above equation for F(x),

$X_T = u + \alpha . (-\ln .\ln (T/(T-1)))$.------------(4)

where T=1/(1-F) is the return period and X_T is corresponding return period value. Here ‘ln’ denotes the natural logarithm.

Equation (4) along with the estimated parameters, has been used in this study for the computation of return period values corresponding to 10, 25, 50, 100, 500 and 1000 years for durations of 1, 2 & 3-day for all 210 stations located inside the Indus basin. Spatial pattern maps for each of the above return periods for the durations of 1-3 days have been prepared which will be useful in designing and planning the hydraulic structures and also for proper management of water resources and flood control studies within the basin. Generalized maps for 10, 100 & 500- year return periods for 1-3 day durations are only shown in Fig.7 (a, b & c) to Fig.9 (a, b & c) to keep the paper concise. From these Figs., it is seen that, 10-year 1-day extreme rainfall values are less than 20 cm in the Indus basin, 500-year 1-day extreme rainfall is of the order of 60cm at a few places, but remains less than 15cm in the northern area of the basin. In case of sub-basin 207, even 500- year value for 3 -day duration remains around 15cm.

The equation for X_T is further used to compute the return periods of PMP for 1-3 day durations. It was found that, for most of the stations in Indus basin, 1-3 day PMP have return periods more than 1000 years, which satisfy the minimum requirement suggested by the Central Water Commission, India (CWC, 1972).

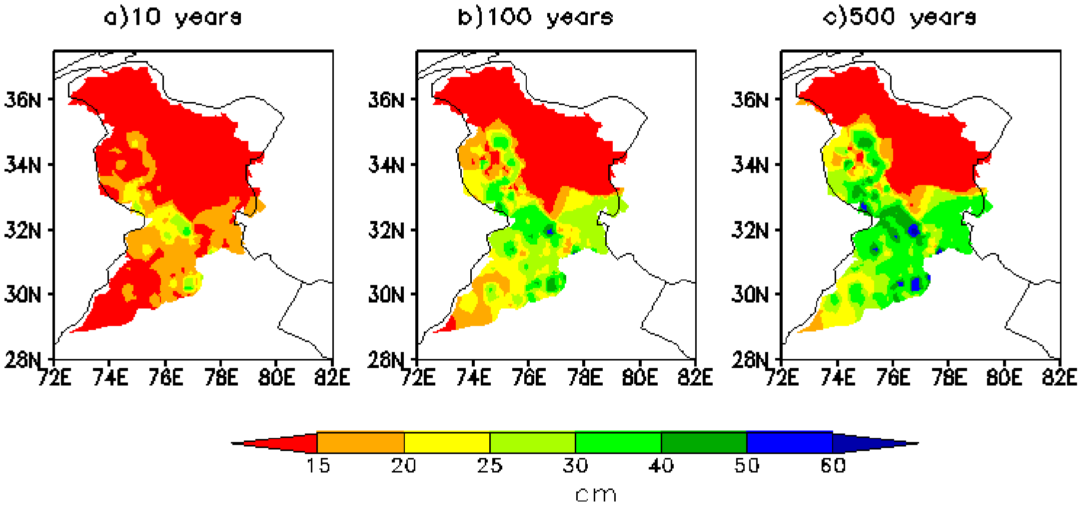


Fig.7 : Spatial patterns of return period rainfall values (cm) for 1-day duration over the Indus basin

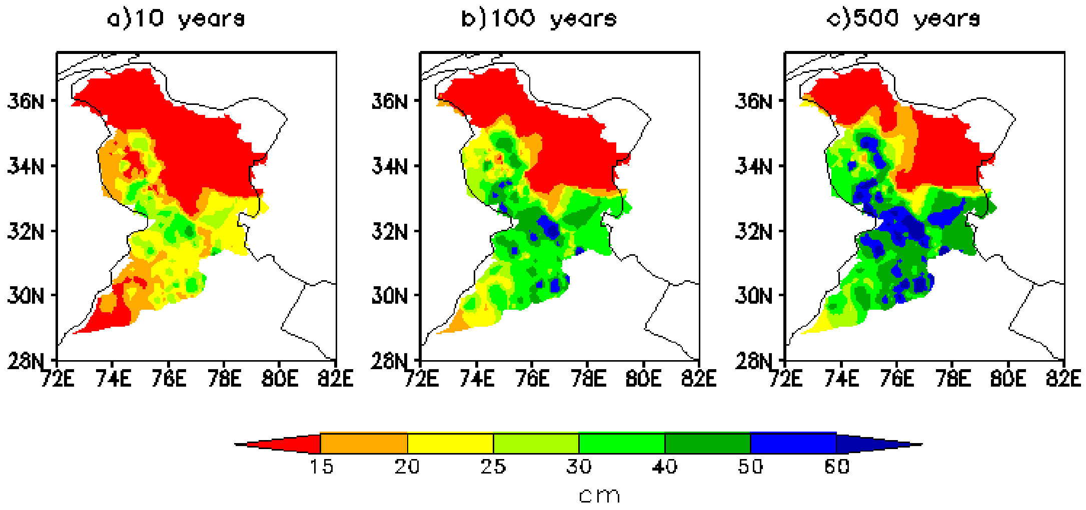


Fig.8 : Spatial patterns of return period rainfall values (cm) for 2-day duration over the Indus basin.

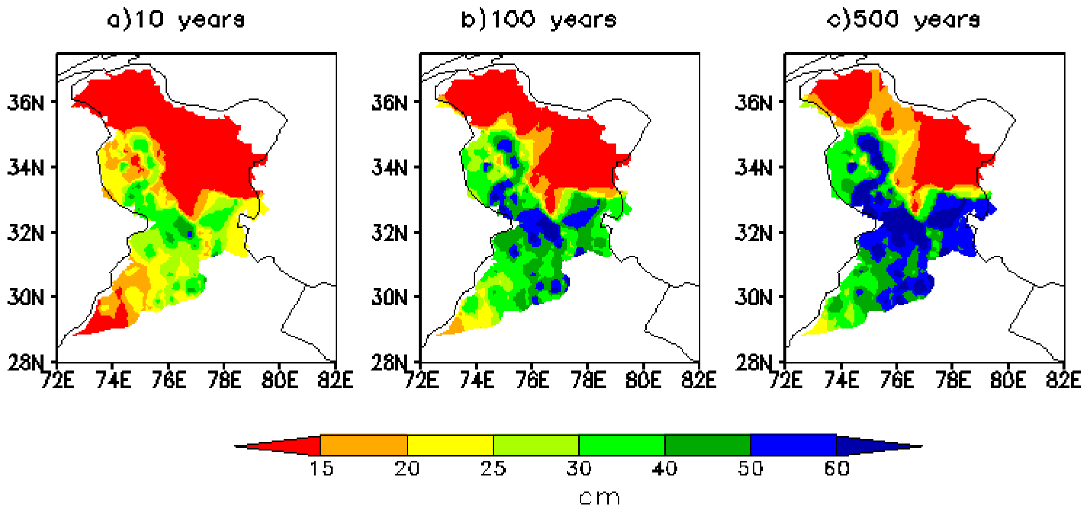


Fig.9 : Spatial patterns of return period rainfall values (cm) for 3-day duration over the Indus basin.

Summary and Conclusions:

Following facts immersed from the present study:

Mean annual rainfall over the basin as a whole has been found to be of the order of 93 cm and southwest monsoon months of July-September contribute 54% to the annual rainfall. Highest ever recorded rainfall over the basin was found to range 2 to 61 cm in 1-day, 3 to 105 cm in 2-day and 4 to 106 cm in 3-day durations respectively. Over the entire basin, point PMP estimates were found to range from about 5 to 98 cm for 1-day, 7 to 137 cm for 2-day and 8 to 163 cm in 3-day durations. Maximum being estimated at Kilba station which is located in sub-basin 201. 1-3 day PMP values for most of the stations within the Indus river basin, have return periods more than 1000 years.

Extreme rainfall, point PMP and maximum rainfall amounts for different return periods over the Indus basin in India presented in this paper will be useful in planning, designing and management of different types of hydraulic structures for optimum utilization of water resources in the basin. Such studies are crucial for the river basins like Indus which is located in highly orographic area where different rain bearing systems like south-west monsoon and western disturbances affect the basin.

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REFERENCES

Bruce, J. P., and Clark, R. H., (1966): Introduction to Hydrometeorology. Pergaman Press, New York.

Central Water Commission (CWC), (1972): Estimation of design flood recommended procedures, N.Delhi.

Dhar, O. N., and Kulkarni, A. K., (1975): Estimation of probable maximum precipitation for some selected stations in and near the Himalayas. Proc. National Symp. on Hydrology. University of Roorkee.

Dhar, O. N., Kulkarni, A. K. and Rakhecha, P. R., (1981): Probable maximum point rainfall estimation for the southern half of the Indian peninsula, Proc. Indian Academy of Sciences., Bangalore, Vol. 90 , No. 1

Dhar, O.N., Mandal, B. N., and Kulkarni, A. K. ,(1987): Some facts about precipitation distribution over the Himalayan region of Uttar Pradesh, Western Himalaya (Environment), Gyandaya Publi., Nainital, U.P. Vol. I

Gumbel, E. J. (1954) : Applied Mathematics Series No . 33, National Bureau of Standards , Washington D. C.

Hershfield, D. M. and Kohlar, M. A. (1960) : An empirical appraisal of the Gumbel extreme value procedure, Journal of Geophys. Res., Vol.65, No. 6.

Hershfield, D. M. (1961) : Estimating the probable maximum precipitation., J. Hydraul. div. Am. soc. Civ. Eng., Vol . 87

Hershfield, D. M. (1962): Proc. American Soc. Civil Engineers, Vol.88,No.Hy6.

Iyer, V. D. and Zafer, M., (1938): Distribution of heavy rainfall of ten inches and above over India, Met.

Dept. Sci. Notes, No. 77.

Khosla, A. N., (1949): Appraisal of Water Resources Analysis and Utilization of data, Proc. United Nations Scientific Conference on conservation and utilization of resources, Sept. 1949

Koutsoyiannis D. (1999) : A probabilistic view of Hershfield's method for estimating probable Maximum Precipitation , Water resour. Res, 35(4), 1313-1322.

Kulkarni, A. K., Mandal, B. N. and Sangam, R. B.,(1991): Analysis of severe rainstorm of 23-25 July, 1989 over northern half of the Indian Peninsula, Trans. Institute of Indian Geographers, Vol 13, No. 1

Mandal, B.N., Deshpande, N.R., Kulkarni, B.D. and Sangam, R.B. (2006): Spatial patterns of highest observed 1 to 3 -day point rainfall over the Indian region, J. of Applied Hydrology, Vol.18, No. 1 & 2.

Myers, V. A., (1967): Meteorological estimation of extreme precipitation for spillway design floods, Tech. Memo. WBTM. Hydro., Vol. 5, U. S. Weather Bureau, Environmental Science Services Administration.

Papalexiou S. M. and Koutsoyiannis D, 2006 : A probabilistic approach to the concept of Probable Maximum Precipitation., advances in Geosciences, 7, 51-54.

Paulhus and Gilman, G. S., (1953) : Evaluation of probable maximum precipitation. Trans. Am. Geophys., Union Vol 34.

Pisharoty, P. R. and Desai, B. N., (1956): Western Disturbances and Indian Weather, Indian Journal of Meteorology and Geophysics, Vol.7.

Rakhecha, P. R., Kulkarni, A. K., Mandal, B. N. and Deshpande, N. R. (1990) : Homogeneous zones of heavy rainfall of 1-day duration over India. Theoretical and Applied Climatology Vol. 41 pp 213 to 219.

Rao, K.L. (1975) : India's Water Wealth, Orient Longman Ltd.,New Delhi, 255pp.

Wiesner, C. J., (1970): Hydrometeorology. Chapman and Hall, Ltd. London.

World Meteorological Organization, (1969): Estimation of maximum floods., Tech. Note 98, WMO, No. 233 TP. 126, TP 126.

World Meteorological Organization, (1970): Guide to hydrometeorological Practices. No. 168, WMO, TP 82.

World Meteorological Organization, (1986): Manual for estimation of probable maximum precipitation, Operational Hydrology Report No. 1 Second Edition, W. M. O. No. 332.