Reverse Problem Formulation for Integrating Process Discharges with Watersheds and Drainage Systems
Managing Phosphorus in Lake Manzala

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Summary
This work introduces a new approach to integrating the discharges of industrial processes with macroscopic watershed systems. The key concept is that environmental quality models (such as material flow analysis) can be inverted and included in an optimization formulation that seeks to determine the maximum allowable target for the process discharges while meeting the overall environmental requirements of the watershed. Because of its holistic nature, this approach simultaneously considers the effects of the inputs and outputs to the watershed (e.g., agricultural, residential, wastewater treatment plants, industrial, and so on) and the various physical, chemical, and biological phenomena occurring within the watershed. An optimization formulation is developed to systematically represent the reverse problem formulation. To illustrate the effectiveness of this approach, a case study is solved to manage phosphorus in Bahr El-Baqar drainage system leading to Lake Manzala in Egypt. The key environmental and economic aspects are addressed and used to screen plant location and discharges.
**Introduction**

Watersheds and drainage systems are macroscopic environmental systems that have a major impact on the Earth's ecology. A watershed covers a geographical region where water flows through various surface and underground drainage pathways. A watershed typically involves a number of tributaries that feed into reaches, streams, or rivers that finally lead to catchment areas such as lakes, seas, and oceans. Watersheds are impacted by surrounding inputs such as agricultural, residential, and industrial systems. At the same time, watersheds have a major impact on the various biological, physical, health, and social components of the surrounding environmental systems. To understand the characteristics of watersheds, how they are impacted by their surroundings, and how they impact their surroundings, it is important to develop quantitative models that track the flows and compositions of various pollutants throughout the watershed. In this regard, a particularly useful tool is material flow analysis (MFA). The key objectives of MFA are to track targeted species and analyze causes for pollution problems in a certain region by accounting for all relevant activities, water sources, water users, pollution sources, and physical/chemical/biological phenomena impacting the watershed. Baccini and Brunner (1991) developed an MFA model for analyzing ecosystems with human activities where mass, energy, and information are being exchanged with the surroundings. Although MFAs are typically used for tracking substances and materials in regions, Lampert and Brunner (1999) used MFA to track principal nutrients discharges, transformation, and flows into the Danube river basin. El-Baz and colleagues (2005) developed an MFA model for watersheds and applied it to the Bahr El-Baqar drainage system for tracking nitrogenous compounds.

It is worth noting that MFA models and other forms of environmental quality models are carried out in the **forward mode**, that is, a given scenario is posed and its consequences are tracked via the MFA. For instance, suppose that a new industrial facility is to be installed in a watershed. The exact location is to be determined. Also, in addition to satisfying the environmental regulations for the process discharges, it is necessary to examine the impact of the process effluents on the watershed. As an illustration, consider the case shown in figure 1. A new industrial facility is to be installed on a watershed. The process has certain process discharges with a given composition of a certain pollutant, $y_{\text{Process}}$. It is important to analyze how the installation of this plant interacts

![Figure 1](image-url)  
*Figure 1* Forward model for tracking discharges of a new process.
with the rest of the watershed (including other sources and users such as agricultural usage and discharge, wastewater treatment plant, residential usage, and discharge, and so on). It is also necessary to assess the impact of the process discharge on the receiving lake. These aspects are also central to any environmental impact assessment of the new plant. In the usual forward mode of environmental assessment, a construction site is selected, and the model is used to track the process discharge throughout the watershed. If the discharge to the lake is acceptable, the environmental discharge of the process is deemed satisfactory. Otherwise, a laborious trial-and-error procedure is adopted to modify the plant design, environmental system, and location until an acceptable discharge to the lake is achieved.

The main theme of this work is to introduce a reverse approach to targeting acceptable discharge of the process while integrating the process effluents with the rest of the watershed. The approach is referred to as reverse problem formulation. This approach starts “with the end in mind.” Therefore, the desired characteristics of the discharge to the lake are first specified. Then, the MFA model is included in an optimization formulation that seeks to determine the maximum acceptable discharge from the process that will have an acceptable impact on the lake while accounting for all interactions throughout the watershed. Figure 2 is a schematic representation of the reverse problem formulation. This work introduces the theoretical formulation for this approach. It also provides an application case study in managing phosphorus compounds in Bahr El-Baqar watershed leading to Lake Manzala in Egypt. First, the basic equations for an MFA model are presented. Then, the optimization formulation for reverse problem formulation is introduced. Next, the case study is discussed. The results of the case study are compared with actual measurements. Finally, the problem of plant location and environmental targets is solved.

**Problem Statement**

Consider a watershed system with its tributaries and reaches. Various input and outputs are associated with the watershed. These include phosphorus (P) in agricultural drainage, treated and untreated wastewater, industrial effluents, and precipitation. System outflows include discharge to lakes and waterways, seepage, and vaporization. Throughout the watershed, there are

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**Figure 2** Reverse approach to integrating the process and the watershed.
chemical and biochemical reactions that lead to transformation of discharged pollutants and formation of by-products. The system surroundings include a given set of wastewater treatment plants, residential, agricultural, and industrial sectors. The catchment of the watershed (e.g., lake) has desired characteristics which define the maximum permissible load of pollutants in the lake (e.g., at the lake outfall). A new industrial facility is to be installed on the watershed. The objective is to select a plant location and to determine the maximum allowable load of discharged pollutants from the plant such that the environmental requirements for the watershed are met.

To address the aforementioned problem, this work provides two new contributions:

1. Development of a reverse problem formulation for the MFA model to determine the maximum allowable target for the process discharge while satisfying the desired environmental performance of the watershed.
2. Application of the devised approach to managing phosphorus ions in Bahr El-Baqr drain system leading to Lake Manzala in Egypt.

**MFA Modeling Equations**

This work adopts the MFA modeling equations developed by El-Baz and colleagues (2005). The key equations are given by:

**Flowrate Balance for the Reaches:**

\[ Q_{i,t} = Q_{i-1,t} + P_{i,t} + D_{i,t} + H_{i,t} + \sum_{j=1}^{N_{trb}} T_{j,i,t} - L_{i,t} - U_{i,t} \]  

(1)

Where

- \( Q_{i,t} \) = Flowrate leaving the \( i^{th} \) reach, m\(^3\)/s
- \( Q_{i-1,t} \) = Flowrate entering the \( i^{th} \) reach, m\(^3\)/s
- \( P_{i,t} \) = Precipitation flow to the \( i^{th} \) reach, m\(^3\)/s
- \( L_{i,t} \) = Net losses from the \( i^{th} \) reach (e.g., seepage, vaporization, use, and so on), m\(^3\)/s
- \( D_{i,t} \) = Nontributary direct discharge to the \( i^{th} \) reach, m\(^3\)/s
- \( H_{i,t} \) = Total discharge to (e.g., industrial discharge + sanitary discharge, and so on) to the reach m\(^3\)/sec
- \( T_{j,i,t} \) = Tributary discharge from the \( j^{th} \) tributary to the \( i^{th} \) reach, m\(^3\)/s
- \( U_{i,t} \) = Usage discharge from the reach \( i^{th} \), m\(^3\)/sec

**Pollutant Balance for the Reaches:**

\[ Q_{i,t} \times C_{Qi,t} = Q_{i-1,t} \times C_{Qi-1,t} + H_{i,t} \times C_{Hi,t} + P_{i,t} \times C_{Pi,t} + D_{i,t} \times C_{Di,t} + \sum_{j=1}^{N_{trb}} T_{j,i,t} \times C_{Tj,i,t} - L_{i,t} \times C_{Li,t} - U_{i,t} \times C_{Ui,t} - \int_{V=0}^{V} r_{i,t} \, dV_{i,t} \]  

(2)

Where

- \( Q_{i,t} \times C_{Qi,t} \) = Load rate leaving the reach via convective flow (where \( C_{Qi,t} \) is the component concentration in the convective stream leaving reach \( i \))
- \( Q_{i-1,t} \times C_{Qi-1,t} \) = Load rate entering the reach via convective flow
- \( H_{i,t} \times CH_{i,t} \) = Total load rate of discharges (e.g., industrial discharge + sanitary discharge, and so on) over reach \( i \)
- \( P_{i,t} \times CP_{i,t} \) = Total load rate of precipitation over reach \( i \)
- \( D_{i,t} \times CD_{i,t} \) = Total load rate of drainage over reach \( i \)
- \( L_{i,t} \times CL_{i,t} \) = Total load of losses (e.g., seepage and vaporization) over reach \( i \)
- \( U_{i,t} \times CU_{i,t} \) = Total load of usage over reach \( i \)

The kinetic expressions may take different forms depending on the system under consideration. Typically, experimental data (including batch experiments) followed by regression analysis provide a convenient method for developing the kinetic functions and accounting for uncertainties. Regardless of the complexity of the kinetic expression, it can always be included in the formulation. For the case of first-order reaction
kinetics, the expression becomes:

\[
\int_{V=0}^{V,t} r_{i,t} \, dV_{i,t} = k_{i,t} \ast CQ_{i,t} \ast V_{i,t} \tag{3}
\]

Where \( k_{i,t} \) is the Arrhenius reaction rate constant. Hence,

\[
Q_{i,t} \ast CQ_{i,t} = Q_{i-1,t} \ast CQ_{i-1,t} + H_{i,t} \ast CH_{i,t} + P_{i,t} \ast CP_{i,t} + D_{i,t} \ast CD_{i,t} + \sum_{j=1}^{N_{trib}} T_{j,it} \ast CT_{j,it} - L_{i,t} \ast CL_{i,t} - U_{i,t} \ast CU_{i,t} - k_{i,t} \ast CQ_{i,t} \ast V dV_{i,t} \tag{4}
\]

Pollutant Balance for the Tributaries

\[
T_{j,it} \ast CT_{j,it} = S^{untreated}_{j,it} \ast CS^{untreated}_{j,it} + S^{treated}_{j,it} \ast CS^{treated}_{j,it} + I_{j,it} \ast CI_{j,it} + P_{j,it} \ast CP_{j,it} + D_{j,it} \ast CD_{j,it} - L_{j,it} \ast CL_{j,it} - U_{j,it} \ast CU_{j,it} - r_{j,it} \ast V_{j,it} \tag{5}
\]

Where

\( S^{untreated}_{j,it} = \) Untreated sewage (sanitary waste) discharged to the \( j^{th} \) tributary, \( m^3/s \)

\( S^{treated}_{j,it} = \) Treated sewage (sanitary waste) discharged to the \( j^{th} \) tributary, \( m^3/s \)

\( I_{j,it} = \) Industrial flow of wastewater discharged to the \( j^{th} \) tributary, \( m^3/s \)

\( P_{j,it} = \) Precipitation flow discharged to the \( j^{th} \) tributary, \( m^3/s \)

\( L_{j,it} = \) Net losses from the \( j^{th} \) tributary (e.g., seepage, vaporization, use, and so on), \( m^3/s \)

\( D_{j,it} = \) Agricultural drainage discharged to the \( j^{th} \) tributary, \( m^3/s \)

To simplify the terminology for modeling the reaches and tributaries, over a given averaged time period these equations are rewritten as:

\[
(Q_u, y_u) = \psi_u \left( Q^{in}_{u}, y^{in}_{u}, I^{in}_{u}, C^{in}_{u} \right) \tag{6}
\]

where \( u \) is an index used for all tributaries and reaches. Also, \( Q_u \) and \( y_u \) are flowrate and composition leaving the \( u^{th} \) reach or tributary. It is described as a function of the vector of all reaches and tributaries feeding into \( u \). These flowrates and compositions are given by the vectors \( Q^{in}_{u} \) and \( y^{in}_{u} \). In addition, the equation accounts for all inputs from the surroundings (e.g., residential, industrial, agricultural) given by the flowrate and composition vectors \( I^{in}_{u} \) and \( C^{in}_{u} \).

Consider a new plant to be installed at location \( \bar{u} \). The flowrate and composition of the process effluent are designated as \( P_{\bar{u}} \) and \( y^{Process}_{\bar{u}} \). Therefore, the MFA model at location \( \bar{u} \) can be modified as follows:

\[
(Q_{\bar{u}}, y_{\bar{u}}) = \psi_{\bar{u}} \left( Q^{in}_{\bar{u}}, y^{in}_{\bar{u}}, I^{in}_{\bar{u}}, C^{in}_{\bar{u}}, P_{\bar{u}}, y^{Process}_{\bar{u}} \right) \tag{7}
\]

This expression can be inverted to have \( y^{Process}_{\bar{u}} \) on the left-hand side, that is

\[
y^{Process}_{\bar{u}} = \Omega_{\bar{u}} \left( Q_{\bar{u}}, y_{\bar{u}}, Q^{in}_{\bar{u}}, y^{in}_{\bar{u}}, I^{in}_{\bar{u}}, C^{in}_{\bar{u}}, P_{\bar{u}} \right) \tag{8}
\]

The desired composition at the outfall is given by \( y^{Outfall, desired} \).

The reverse problem formulation seeks to find the maximum allowable composition of the process effluent while including the modeling equations for the whole watershed using the MFA and the inverted equation relating the process effluent to the various inputs and outputs while satisfying the discharge targets at the outfall. The reverse problem formulation is described by the following optimization model:

Maximize \( y^{Process}_{\bar{u}} \) \hspace{1cm} \tag{9}

Subject to:

Watershed model:

\[
(Q_u, y_u) = \psi_u \left( Q^{in}_{u}, y^{in}_{u}, I^{in}_{u}, C^{in}_{u} \right) \quad \forall u \neq \bar{u} \tag{10}
\]

Process model:

\[
y^{Process}_{\bar{u}} = \Omega_{\bar{u}} \left( Q_{\bar{u}}, y_{\bar{u}}, Q^{in}_{\bar{u}}, y^{in}_{\bar{u}}, I^{in}_{\bar{u}}, C^{in}_{\bar{u}}, P_{\bar{u}} \right) \tag{11}
\]

Outfall regulation:

\[
y^{Outfall, desired} \leq y^{Outfall} \tag{12}
\]
Process regulations, which may be in the form of a concentration limit or a load limit, that is

\[
\gamma_u^{\text{Process}} \leq \gamma_u^{\text{Desired}} \quad (13a)
\]

or

\[
Q_u \cdot \gamma_u^{\text{Process}} \leq \text{Load}_{\text{Desired}}^u \quad (13b)
\]

Constraints 13a and 13b ensure that the pollution levels are maintained within an acceptable limit at selected locations. These locations may include all discharge and monitoring points (i.e., all \(u\)’s) or may be designated for selected locations. It is also worth noting that such constraints may be imposed in the form of concentration limits of pollutants (constraint 13a) or in the form of load of pollutant (constraint 13b).

The solution of the aforementioned program determines the target composition for the maximum allowable process effluent and provides the revised data for the watershed reaches and tributaries as a result of placing the plant at location \(u\). It is also worth noting that the formulation allows in-plant modification when the function \(\Omega_\mu\) is expressed in terms of the design and operating variables of process \(\mu\). In addition, if desired, in-process modifications of other processes and inputs may be accounted for by adding process models with the allowable degrees of freedom for the various processes:

\[
(Q_u, \gamma_u) = \psi_u \left( Q_{u}^{\text{in}}, \gamma_u^{\text{in}}, I_u^{\text{in}}, C_u^{\text{in}} \right) \quad \forall u \quad (14)
\]

This approach fosters innovative designs because the target is determined ahead of detailed design and provides the designer with an overall target within which it is possible to generate numerous feasible alternatives. Finally, it is important to note that the aforementioned analysis and optimization can be adopted as a decision-support tool over time. When new plants are being considered, the analysis is redone with the new data. Also, when different scenarios of candidate new plants are considered, the optimization formulation can be solved to compare the various alternatives and provide a systematic method for comparison and decision making.

### Case Study: Managing Phosphorus in Bahr El-Baqar Drainage System and Lake Manzala

**Overview of Bahr El-Baqar Drainage System**

Bahr El-Baqar drain is one of the largest drains east of the Nile Delta in Egypt. The starting point originates from the intersection point of Qalyoubia drain and Belbies drain that start from the northern part of greater Cairo. Its end point is the inlet of the southern part of Lake Manzala. The drain has a length of 106.5 kilometers (km) and with a bed width ranged from 23 meters (m) at the start point and up to 70 m at the end point, with a side slope of 2:1.

The drain receives several types of discharges including agricultural drainage, treated and untreated domestic wastewater, and industrial wastewater. The agricultural drainage and the domestic wastewater are considered the two major sources of discharges contributing significantly to the flow in the drain. The industrial wastewater is primarily discharged from the industrial zone of Shoubra El-Khima in Cairo through Sheben El-Qanater drain. The discharges from agricultural drainage are received from the surrounding areas. Along the drains are some drainage pump stations established by the Egyptian Ministry of Water Resources and Irrigation to collect water from the drains network and raise it by pumping stations into Bahr El-Baqar drain. Two types of reuse from the drain exist: official and unofficial. The official reuse is regulated by the Egyptian Ministry of Water Resources and Irrigation to collect water from the drains network and raise it by pumping stations into Bahr El-Baqar drain. Two types of reuse from the drain exist: official and unofficial. The official reuse is practiced by farmers along Bahr El-Baqar watershed. As the domestic and the agricultural drainage are the main contributors to the flow in the drain they will be described as follows:

**Domestic Wastewater**

The Belbeis and Qalyoubia drains are the two major drains carrying the wastewater from Cairo discharge to Bahr El-Baqar drain. The Belbies drain receives wastewater from two wastewater treatment plants east of Cairo; El-Gabal El-Asfar
and El-Berka. In addition, domestic wastewater is discharged from urban as well as rural areas along the drain. Conversely, Qalyoubia drain carries wastewater from Shoubra El-Kheima via Balqas wastewater treatment plant in addition to the wastewater from the urban as well as rural areas along the drain.

**Agricultural Drainage**

Three catchments of agricultural areas contribute to the agricultural drainage runoff in Bahr El-Baqar drain. Qalyoubia catchment’s area serves almost 299,200 acres,\(^3\) Belbies drain serves an area of almost 93,400 acres, and Bahr El-Baqar drain is estimated to serve an area of almost 639,500 acres. The agricultural drainage contributing to the flow in the main drains is caused by either the tributaries or direct discharge to these drains. Vegetables are the dominant plants along the Qalyoubia drains near Cairo while other crops such as rice, wheat, maize, alfalfa, and so on are distributed along the watershed. Traditional crops such as rice, wheat, cotton, alfalfa, and so on are the dominant plants along both Belbies and Bahr El-Baqar drain in addition to the vegetables.

**Phosphorus Transformations in Water Systems**

Phosphorus is a source of concern for the water quality in Lake Manzala. Excessive presence of phosphorus leads to eutrophication of the water ecosystem. Eutrophication (Garnier et al., 2005; Jarvie et al., 2006; Malmaeus et al., 2006) promotes the growth of phytoplankton and other algal plants, leading to increased water pollution, which can have detrimental impacts in the marine system such as

1. Increased phytoplankton/algal plant growth and decay
2. Depletion of dissolved oxygen
3. Reduction of water quality
4. Decrease in resource value of rivers, lakes, and estuaries, such as death of aquatic life (which in turn negatively impacts recreational activities)
5. Impact in taste, odor, and water treatment

Phosphorus enters the water pool via point and nonpoint sources (e.g., Bowes et al., 2008; De Laender et al., 2007). Point sources are those that can be quantified and attributed to one source, such as urban effluent from wastewater treatment plants. Nonpoint sources, such as soil erosion, are those that cannot be easily attributed to one source and are more difficult to quantify. Table 1 summarizes the sources of phosphorus entering the drainage system.

In water, phosphorus is present in dissolved and particulate forms as a result of numerous biological and chemical transformations. Each form contains both organic and inorganic phosphorus. However, only dissolved forms can be taken up by algae and phytoplankton, which are the prime causes for eutrophication. The dissolved forms are usually characterized as dissolved inorganic phosphorus (DIP) and, more generally, soluble reactive phosphorus (SRP). Key steps (Malmaeus et al., 2006; Maurer and Boller, 1999; Nausch et al., 2004; Paytan and McLaughlin, 2007; Ruley and Rusch, 2004; Zhou et al., 2007) in phosphorus transformation are as follows:

Table 1 Sources of phosphorus entering the drainage system

<table>
<thead>
<tr>
<th>Sources of phosphorus</th>
<th>Point sources</th>
<th>Nonpoint sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wastewater effluent</td>
<td>Runoff from agriculture</td>
<td>Runoff from pastures/ranges</td>
</tr>
<tr>
<td>Runoff/leachate from waste disposal systems</td>
<td>Runoff from agriculture</td>
<td>Runoff from pastures/ranges</td>
</tr>
<tr>
<td>Runoff from animal feedlot</td>
<td>Runoff from untreated industry effluent</td>
<td>Runoff from sewage systems</td>
</tr>
<tr>
<td>Runoff from storm and sanitary sewers</td>
<td>Runoff from treated industry effluent</td>
<td>Runoff from sewage systems</td>
</tr>
<tr>
<td>Runoff from construction sites</td>
<td>Runoff from treated industry effluent</td>
<td>Runoff from sewage systems</td>
</tr>
</tbody>
</table>

1. Increased phytoplankton/algal plant growth and decay
2. Depletion of dissolved oxygen
3. Reduction of water quality
4. Decrease in resource value of rivers, lakes, and estuaries, such as death of aquatic life (which in turn negatively impacts recreational activities)
5. Impact in taste, odor, and water treatment

1. Photosynthetic uptake
2. Excretion
3. Chemical transformation
4. Hydrolysis of organic phosphorus
5. Decomposition of particulate organic phosphorus
6. Sediment decomposition

The following section provides more information on these steps.

**Photosynthesis (SRP Uptake)**
In photosynthesis, O2 is released by phytoplankton and dissolved inorganic phosphorus is taken up through the following reaction:

\[
106 \text{CO}_2 + 64 \text{H}_2\text{O} + 16 \text{NH}_3 + \text{H}_3\text{PO}_4 \rightarrow \text{C}_{106}\text{H}_{179}\text{O}_{68}\text{N}_{16}\text{P} + 106 \text{O}_2
\]

**Respiration (SRP Release)**
DIP is released into the water column through respiration of phytoplankton. During respiration, phytoplankton utilize O2 and release CO2.

\[
\text{C}_{106}\text{H}_{179}\text{O}_{68}\text{N}_{16}\text{P} + 106 \text{O}_2 \rightarrow 106 \text{CO}_2 + 64 \text{H}_2\text{O} + 16 \text{NH}_3 + \text{H}_3\text{PO}_4
\]

**Decomposition of Particulate Organic Phosphorus and Hydrolysis of Dissolved Organic Phosphorus (Conversion to SRP)**
Organic phosphorus (OP), which consists of living and dead phytoplankton, phosphorus precipitates and phosphorus adsorbed onto particulates, is not directly available for uptake into phytoplankton. It must first undergo hydrolysis, which converts dissolved organic phosphorus (DOP) to DIP. It should be recalled that organic phosphorus consists of particulate organic phosphorus (POP) and DOP. POP is converted to DOP through decomposition, which then undergoes hydrolysis to DIP.

\[
\text{POP} = \text{Plant death} + \text{POP settling} \\
- \text{decomposition of organic matter}
\]

\[
\text{DOP} = \text{decomposition of POP} \\
- \text{hydrolysis of DOP} \\
+ \text{plant excretion & death} \\
+ \text{dissolved sediment flux}
\]

**Sedimentation**
Both organic and inorganic phosphorus can move between the water column and the sediment through adsorption/desorption onto particulate matter. In addition, decomposition organic matter (plant) increases organic phosphorus.

**Development of the MFA Model for Bahr El-Baqar Drainage System**
In developing the MFA mathematical model, the following assumptions were invoked for the numerical data of some elements of the system:

1. Negligible rain: The precipitation flow value (P) was neglected according to the nature of weather in Egypt.
2. Negligible losses: The losses (L) including seepage, and vaporization were assumed to be much smaller than the convective flow.
3. The concentration of the phosphorus in the effluent from the waste water treatment plant (WWTP) was taken to be 9 milligrams per liter (mg/l). This assumption is based on the laboratory analysis of different WWTP in Egypt such as El Gabal el Asfar and Zenein WWTP.
4. The phosphorus concentration in the untreated sewage (SU) in rural Egypt was taken to be 15 mg/l.
5. The phosphorus concentration for wastewater primary treated was taken as 9.75 mg/l as this value considers the removal of phosphorus concentration after secondary stage to be approximately 35%.
6. The agricultural drainage concentration (CD) was taken as 1.5 mg/l.

**MFA Model Validation and Results**
Based on equations 1–5, the MFA model was developed for all tributaries and reaches in Bahr El-Baqar drainage system. Only one fitting parameter was used that represents the kinetic rate constant. It was evaluated to be 0.78 day\(^{-1}\) by minimizing the error squared (of the model prediction versus the measured value) of phosphorus concentration at the Lake Manzala outfall. The model was coded using the software LINGO (Schrage, 2001). Next, the model results were compared with measurements of phosphorus compositions over nine monitoring stations. These monitoring stations are shown in figure 3.
Figure 3 Schematic representation of Bahr El-Baqr drainage system and Lake Manzala. Monitoring stations are indicated by the designation EB (engineering base) followed by the station number. WWTP = waste water treatment plant.
Figure 4 represents the model results of phosphorus concentration versus the single measurements at the monitoring station. The results show good agreement between the measurements and the calculations at most of the monitoring stations. Deviations at monitoring stations EB4, EB5, and EB7 may be attributed to measurement inaccuracies due to strong hydrodynamic mixing at these locations.

**Reverse Problem Formulation**

Next in the case study, we consider the reverse problem formulation for the environmental impact of an industrial process. A new fertilizer plant is to be built in one of four possible locations discharging into Bahr El-Baqar drain. As part of the environmental impact assessment, it is required to determine whether or not the process effluents will have an acceptable impact on phosphorus discharge into Lake Manzala. Also, given that different plant locations will have different environmental impact and may entail different treatment costs, the company building the facility is interested in comparing the costs associated with the four candidate locations. Sites I and II are located on Bahr El Baqar drain, Site III is located on Belbeis drain, and Site IV is on the Qalyoubia drain. Figure 5 shows the locations of the four candidate sites.

The fertilizer plant will discharge an effluent with a flowrate of 2.0 cubic meters per second (m$^3$/s) containing a phosphorus concentration of 12.5 parts per million (ppm). To prevent eutrophication, the phosphorus input to Lake Manzala cannot exceed 1.3 ppm. As a result, any potential location that discharges phosphorus that leads to an increase in the final watershed input above the allowable concentration will need to employ interception and treatment (phosphorus removal unit) at a cost of $21/kg of P removed. Two costs are to be considered for comparison: phosphorus treatment cost (using interception devices to lower phosphorus composition to an acceptable level) and cost of land. The land cost (annualized over the useful life period of the project) for each site is listed in table 2. Thus, it is desired to determine the optimal site location for the
Figure 5  Candidate site locations for the fertilizer plant.
Table 2  Treatment cost for the four candidate plant locations

<table>
<thead>
<tr>
<th>Candidate location</th>
<th>Land cost $ MM/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site I</td>
<td>10</td>
</tr>
<tr>
<td>Site II</td>
<td>17</td>
</tr>
<tr>
<td>Site III</td>
<td>35</td>
</tr>
<tr>
<td>Site IV</td>
<td>18</td>
</tr>
</tbody>
</table>

Note: $MM/yr = millions of dollars per year.

The results indicate that it is possible to have a discharge of 18.73 mg/l for Site IV and still meet the watershed concentration of 1.3 mg/l. Because the plant will not discharge more than the design concentration of 12.5 mg/l, the actual concentration of the final input to the lake will be 1.26 mg/l. As a result of the long residence time and the natural phosphorus removal mechanism in the reach, the effluent from Site IV will reduce the total P load to the lake.

Conclusions

A new approach has been introduced for integrating discharges of industrial processes with the larger watershed system to which they discharge. The concept of reverse problem formulation has been introduced to enable the identification of maximum allowable target compositions of pollutants in the process effluents that upon interaction with the rest of the watershed will meet the overall environmental requirements of the system. An optimization formulation has been developed to systematically implement the reverse problem concept. A case study on managing phosphorus in Bahr El-Baqar has been solved. First, an MFA model was developed and verified via comparison with measurements. Next, the problem of locating a new fertilizer plant has been addressed. The reverse problem formulation has been used to determine target compositions for phosphorus corresponding to four candidate locations. Economic evaluations were used to select a site.

The devised approach and associated mathematical formulation provide a very powerful and

Table 3  Results of reverse problem formulation and associated economics

<table>
<thead>
<tr>
<th>Location</th>
<th>Maximum target for P in effluent mg/L (calculated through reverse problem formulation)</th>
<th>Minimum P load (kg/yr) to be removed to satisfy the desired target for Lake Manzala</th>
<th>Interception cost, $MM/yr</th>
<th>Total annualized cost $MM/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site I</td>
<td>5.6</td>
<td>435,384</td>
<td>9.14</td>
<td>19.14</td>
</tr>
<tr>
<td>Site II</td>
<td>10.21</td>
<td>144,387</td>
<td>3.03</td>
<td>20.03</td>
</tr>
<tr>
<td>Site III</td>
<td>13.93</td>
<td>0</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>Site IV</td>
<td>18.73</td>
<td>0</td>
<td>0</td>
<td>18</td>
</tr>
</tbody>
</table>

Note: $MM/yr = millions of dollars per year
generally applicable framework that offers the following advantages:

- Overcoming the conventional limitation of the forward modeling approaches, which require laborious trials to adjust the environmental performance of the process and to study its impact on the macroscopic environmental systems.
- Incorporating different forms of watershed models in the same overall optimization framework.
- Determining rigorous targets for maximum allowable discharges (pollutant concentration and load) of a process such that the environmental constraints are met at any location in the watershed/drainage system. These targets are determined without commitment to the specific design changes in the plant. This allows numerous innovative design modifications to be developed.

Acknowledgements

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Notes

1. One kilogram (kg, SI) ≈ 2.204 pounds (lb).
2. One meter (m, SI) ≈ 3.28 feet (ft).
3. One acre ≈ 0.405 hectares (ha) ≈ 4.05 × 10⁻³ square kilometers (km², SI) ≈ 1.56 × 10⁻³ square miles.
4. One milligram (mg, SI) = 10⁻³ grams (g) ≈ 3.53 × 10⁻⁵ ounces (oz). One liter (L) = 0.001 cubic meters (m³, SI) ≈ 0.264 gallons (gal).

References


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