Selection of Suitable Aggregation Function for Estimation of Aggregate Pollution Index for River Ganges in India

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Abstract: The present study aims to select the most appropriate aggregation function for estimation of the Ganga River pollution index (GRPI). Following the Delphi technique based on expert opinion, 16 water pollutant variables are selected; the weights of each pollutant variable based on their relative significance are determined, and the average subindex curves for each variable are drawn. Using the weights, average parameter's value and the corresponding subindex value, 18 different aggregation functions are tested and analyzed. Literature reveals that most aggregation methods suffer from ambiguity and eclipsing problems due to faulty selection of aggregation function. From the results of the present analysis, 12 aggregation functions are screened out on the basis of ambiguity and eclipsing, constant functional behavior, and nonaccountability of weights in functions criteria. Finally, the remaining 6 aggregation functions are subjected to sensitivity analysis. From the results of sensitivity analysis, it is concluded that the weighted arithmetic mean function, being a true linear, least ambiguous and eclipsing free function, is the most representative aggregation function for estimation of GRPI for River Ganges.

DOI: 10.1061/(ASCE)0733-9372(2008)134:8(689)

CE Database subject headings: Aggregates; Rivers; Water pollution; India.

Introduction

The evaluation of water quality in developing countries has become a critical issue in recent years, especially due to the concern that fresh water will be a scarce source in the future. Water quality can be studied scientifically if an accurate estimate of water quality is available in the form of an index. However, the overall composite water quality is sometimes difficult to evaluate from a large number of quality variables. Further, the present conditions in the manual data processing of the large number of analytical data practically prevents faster interpretation of the results so that many attempts were made to present them in more understandable and acceptable ways using water quality indices (WQI) or using river pollution indices (RPI).

To describe water quality based on the pollution load, it is useful to employ a subindex of a quality variable to express the pollution level of water bodies on a 0 (best quality nil pollution) to 100 (worst quality severe pollution) scale. In this context, a variety of subindices have been developed and proposed in the past. In order to evaluate the changes in the Ganga River water quality due to a combined influence of several water quality variables, it was deemed necessary to identify a suitable indexing system. A technique to quantify the pollution potential of River Ganges in India on a comparative scale is developed using an appropriate aggregate water pollution index known as the Ganga River pollution index (GRPI). GRPI is a single number that expresses the water pollution condition of River Ganges by integrating measurements of 16 water quality variables and provide a simple and concise method for expressing the water quality of River Ganges for general uses. Although, the aggregate indices describe the water quality by accounting the impact of various quality variables, the public interest is based entirely on the aesthetic aspects of a water environment (House 1996).

Over the last three decades, a number of mathematical functions for aggregation of water quality and water pollution indices have been suggested (Horton 1965; Brown et al. 1970; Prati et al. 1971; Dinius 1972; Dee et al. 1973; McDuffie and Haney 1973; Inhaber 1974; Walski and Parker 1974; Truett et al. 1975; Landwehr and Deininger 1976; Ott 1978; Stoner 1978; Bhargava 1983; Smith 1989, 1990; Gijanovic 1999; Pesce and Wunderlin 2000; Swamee and Tyagi 2000; Cude 2001; Bardalo et al. 2001; Nagels et al. 2002; Said et al. 2004; Swamee and Tyagi 2007). However, several of the mathematical forms give misleading results in certain circumstances.

There are four basic steps primarily involved in water pollution index design: (1) selection of key water pollutant variables; (2) determination of weight for each selected variables; (3) formulation of subindices curves or their mathematical functions; and (4) aggregation of the subindices to yield an overall aggregate index. Among these, the aggregation process is the most impor-

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Note. Discussion open until January 1, 2009. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on November 14, 2006; approved on January 14, 2008. This paper is part of the ***Journal of Environmental Engineering***, Vol. 134, No. 8, August 1, 2008. ©ASCE, ISSN 0733-9372/2008/8-689–701/$25.00.

tant step and hence, the present study aims to search a suitable aggregation function for GRPI among various available mathematical functions in the literature.

Swamee and Tyagi (2007) have recently mentioned that most aggregation methods suffer from three shortcomings: ambiguity, eclipsing, and rigidity. Ambiguity (overestimation) problems exist where the aggregate index is too high and crosses a critical level. The ambiguity free aggregation function is the one in which, if all but one variable, is of acceptable quality. The acceptable quality variable should not influence the aggregation process and the aggregation should reflect the subindex of the impaired quality variable (Ott 1978; Swamee and Tyagi 2000, 2007). Eclipsing (underestimation) problems exist when the aggregate index fails to reflect poor water quality of one or more water quality variables. The eclipsing free aggregation should be biased toward a poor quality subindex. Further, the eclipsing problem progressively worsens with an increase in the number of quality variables (Swamee and Tyagi 2000). Rigidity problems exist when additional variables are included in the index to address specific water quality concerns, but the faulty aggregation function might artificially reduce the value of the aggregate index, such that it does not accurately reflect the true water quality. The most suitable aggregation function is the one, which is either free from these problems or it should minimize these effects in the aggregate index. Besides, some aggregation functions are insensitive and exhibit a constant nature with respect to variations in the subindices of the water pollutant variables, e.g., minimum and maximum operator functions, etc. Further, all the variables do not possess equal significance with regards to the contribution toward water pollution. Therefore, the accounting of their significance levels/weighting factors in aggregation function is necessarily being considered (Ott 1978; Kumar and Alappat 2004). Also, the aggregation function should possess a high sensitivity to the changes in subindices of the selected water pollutant variables (Kumar and Alappat 2004).

Keeping the previous facts in mind, in this paper, 16 variables (selected from expert opinion) were monitored fortnightly for a consecutive period of two-years (January 2004 to February 2006) at 12 different sampling stations between the city of Allahabad and Varanasi, covering a stretch of about 185 km on the course of River Ganges. The methodology based on Delphi technique in the formulation of subindices and subsequently the GRPI is described in brief. The aggregated GRPI have been computed using 18 different aggregation functions available from the literature. The various functions are subjected to different short-listing criteria such as ambiguity and eclipsing criteria, constant functional behavior, nonaccountability of weight in functions criteria followed by a sensitivity analysis of prescreened functions. The aggregation function so selected is intended for suitability-for-use descriptions of River Ganges water quality in a simple manner.

Materials and Methods

Twelve monitoring stations were selected between the city of Allahabad and Varanasi in the State of Uttar Pradesh, India, covering a stretch of 185 km (Fig. 1). A brief description of only five of the monitoring stations (1, 4, 7, 8, and 10) is presented here.

Sampling Station 1 (Mahaveerpuri) is located 11.5 km upstream of the Sangam (Station 4, a point of confluence of River Ganges and Yamuna in the city of Allahabad) and is selected at about 0.75 km downstream of a major city drain discharging the untreated sewage effluent into the river. Station 4 is selected at Sangam about 0.15 km from right bank of River Ganges, where millions of devotees take holy dip especially during festivals like "Kumbh" and "Magh Mela." Besides, a number of religious people also take holy dips at Sangam everyday. Thus, Station 4 is quite significant from both religious and pollution points of view. Station 7 (Lavion kala) is located 5.0 km downstream of Station 4 and is selected downstream of an open drain discharging untreated industrial wastes and sewage into the river at a distance of 0.6 km from Station 7. Station 8 (Baria ghat) is located approximately 108 km downstream of Sangam and just upstream of city of Mirzapur. Station 10 (Rajendra Prasad bathing ghat) is located in the city of Varanasi at about 171.5 km downstream of Sangam. Station 10 is also significantly polluted due to municipal wastewater discharges from densely populated city of Varanasi.

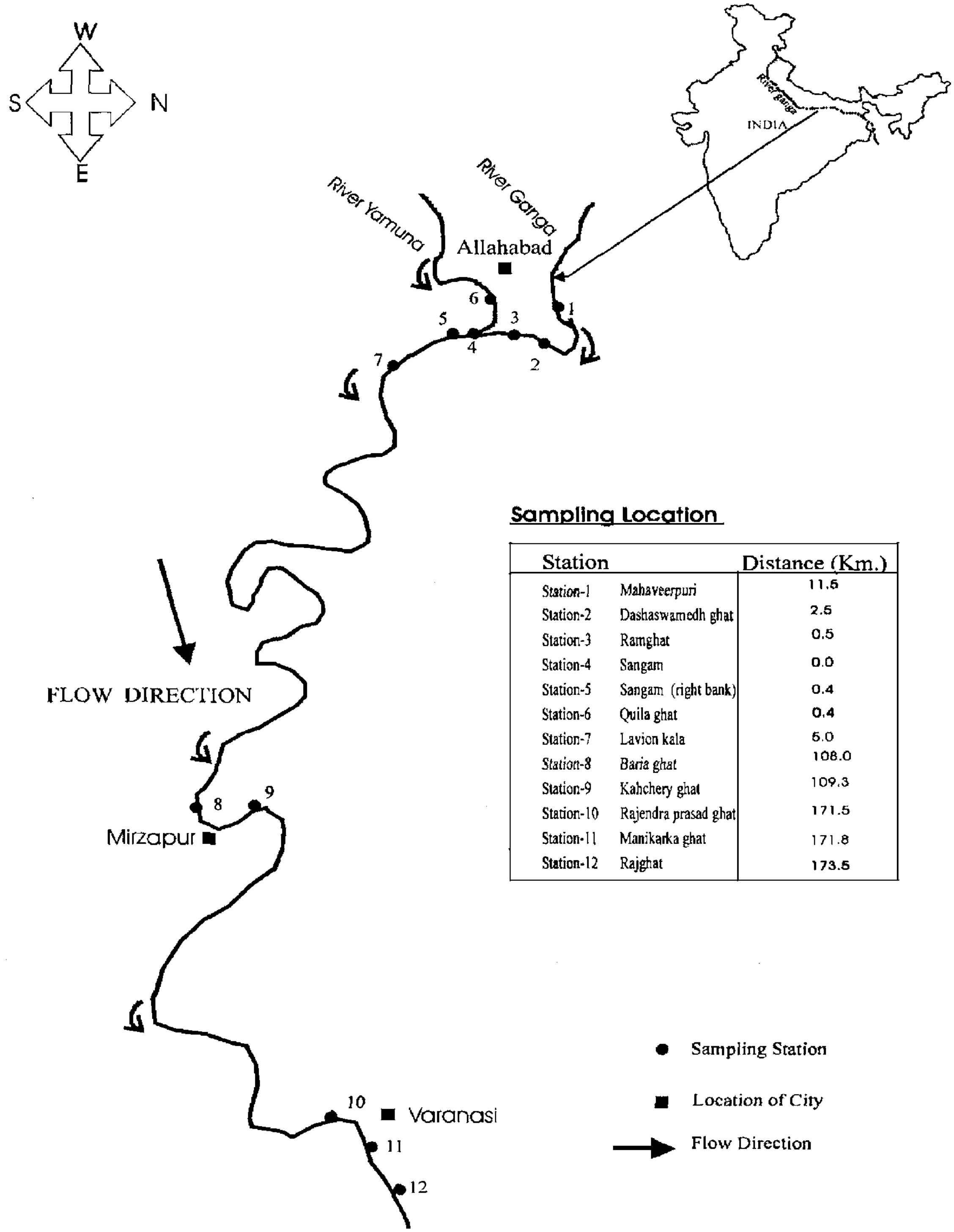
The water samples were collected during a two-year period (January 2004 to February 2006) and all the selected parameters were monitored fortnightly except the heavy metals, which were monitored quarterly. The samples were collected simultaneously from all the 12 stations across the width of the river at a distance of about one-third its width from the river bank and at a depth of 30–40 cm below the water surface. The samples collected were then transported to the environmental engineering laboratory of the Motilal Nehru National Institute of Technology at Allahabad, Uttar Pradesh, India and all 16 parameters were analyzed as per the procedure laid down in Standard Methods (APHA et al. 1995).

Methodology for Formulation of GRPI

Following the Delphi technique (a technique based on the expert's opinion), 130 panelists including 50 academicians, 20 consulting engineers, 40 regulatory officers from various departments of Uttar Pradesh State Government; 10 medical officers, 5 managers of public utilit, and 5 Nongovernment organizations were surveyed. Out of 130 panelists only 70 have responded. A panel of 70 experts was involved in the process of replying to the questionnaires. Three questionnaires were sent to the panelists with an aim of arriving at a consensus selection of pollutant variables, their weights, and the development of subindices curves. A detailed description on theory and development of subindices is available elsewhere in literature (Ott 1978; Smith 1990; Kumar and Alappat 2004). The primary steps followed in the formulation of GRPI are briefly summarized here.

A list of 54 water pollution parameters including physical, chemical, heavy metals, and biological reflecting the condition of water quality was sent to the experts for possible inclusion in the formulation of GRPI. In Questionnaire 1, the panelists were briefed about the possibility of preparing a tool in the form of the GRPI, and its subsequent applications in quality description of River Ganges for different uses. Panelists were further requested to rate each variable according to their increasing significance level on a scale from 1 to 5. Based on the response received from the panelists to Questionnaire 1, and the subsequent questionnaire, 2, which was aimed to arrive at a better consensus, 16 water pollutant variables were selected for their inclusion in the formulation of GRPI.

Different water quality variables possess different significance levels to overall water quality at different times and locations (Bhargava 1983; Cude 2001). The choice of weights is often a source of controversy too. A logical and defensible weighting scheme could improve the interpretability and credibility of the index (Ott 1978). A list of 16 selected variables along with their



Station		Distance (Km.)
Station-1	Mahaveerpuri	11.5
Station-2	Dashaswamedh ghat	2.5
Station-3	Ramghat	0.5
Station-4	Sangam	0.0
Station-5	Sangam (right bank)	0.4
Station-6	Quila ghat	0.4
Station-7	Lavion kala	5.0
Station-8	Baria ghat	108.0
Station-9	Kahchery ghat	109.3
Station-10	Rajendra prasad ghat	171.5
Station-11	Manikarka ghat	171.8
Station-12	Rajghat	173.5

Fig. 1. Sampling stations and their locations on course of River Ganges between the cities of Allahabad and Varanasi

significance levels is presented in Table 1. As all the pollutant variables received different significance levels from experts, the variables must have different weights. For deriving the weights, the arithmetic sum of the significance ratings for all the selected variables was calculated and each variable was given a weight in proportion to the significance value it obtained on a scale of 1, so that the total weight of all the pollutant variables is unity.

In the third questionnaire, the panelists were requested to develop the rating curves for all 16 selected variables on marked graph sheets with levels of river water pollution (subindex score) from 0 to 100 indicated on the ordinate of each graph, whereas various level of concentrations of the particular variable, up to the maximum limits reported in literature, were marked along the abscissa. The panelists were requested to draw a curve on each graph, which, in their judgment, represented the river water pollution produced by the various concentrations of each pollutant variable. The panelists were requested to start the curves from a minimum subindex score of 5 for each of the pollutants, even if there is no contamination from the pollutant to the overall river water pollution. Therefore, the theoretical range of subindex rating is selected from 5 to 100. The responses received on the graph sheets from the panelists were used to produce a set of "average

Table 1. Significance Level, Weight, Average Concentration, and Subindex Value of Pollutant Variables at Different Monitoring Stations

Parameters	Significance	Pollutant weight	Station 1		Station 4		Station 7		Station 8		Station 10	
		W_i	C_i	P_i	C_i	P_i	C_i	P_i	C_i	P_i	C_i	P_i
Chromium (mg/L)	4.15	0.079	0.02	25.00	0.01	10.00	0.03	35.00	0.01	10.00	0.01	10.00
BOD_5 (mg/L)	4.05	0.077	5.88	65.00	4.95	61.00	4.16	56.00	3.15	49.50	3.27	50.50
Lead (mg/L)	4.05	0.077	0.02	46.00	0.01	26.00	0.03	55.00	0.01	26.00	0.02	46.00
MPN/100 mL	4.00	0.076	7416.67	87.50	13016.67	90.00	5150.00	86.00	2959.26	82.50	9198.15	88.00
Dissolved oxygen (mg/L)	3.90	0.074	7.80	24.00	7.67	24.50	7.30	26.00	7.53	25.50	7.43	25.50
pH	3.80	0.072	8.32	17.50	8.31	17.50	8.27	17.00	8.11	16.00	8.14	16.00
Chloride (mg/L)	3.30	0.063	58.76	17.50	67.56	18.50	51.06	16.00	49.83	16.00	49.74	16.00
Copper (mg/L)	3.15	0.060	0.09	7.00	0.20	7.50	0.05	6.50	0.08	7.00	0.11	7.50
Total phosphorus (mg/L)	3.10	0.059	0.63	28.00	0.40	20.00	0.61	28.00	0.45	25.00	0.44	20.00
Zinc (mg/L)	3.05	0.058	0.18	8.00	0.17	8.00	0.19	8.00	0.06	7.50	0.15	8.00
Total solids (mg/L)	3.00	0.057	647.30	28.00	635.67	27.50	497.41	25.00	493.07	25.00	500.19	25.00
Alkalinity (mg/L)	2.80	0.053	197.57	45.00	191.59	43.00	199.67	46.00	195.15	45.50	198.44	46.00
Hardness (mg/L)	2.65	0.050	159.17	24.50	149.50	23.50	154.85	24.00	156.33	24.00	154.93	24.00
Turbidity (NTU)	2.60	0.049	96.89	90.50	70.74	83.00	56.26	85.00	71.87	87.50	70.35	82.00
TKN (mg/L)	2.55	0.049	0.91	9.00	1.01	10.00	1.09	11.00	1.05	10.00	0.94	9.00
Sulfate (mg/L)	2.45	0.047	27.00	7.50	33.03	8.00	30.52	8.00	19.57	6.00	26.94	7.50
Total	52.60	1.000										

Note: W_i=weight for ith parameter; C_i=average concentration or value for ith parameter; and P_i=pollution subindex for ith parameter.

curves," one for each pollutant variable. The resulting subindex rating curves are shown in Figs. 2 (a–p). In each panel, the bold line shows the average subindex score of all the panelists' curves. The calculated weight (W_i) for each pollutant variable, two year's average concentration (C_i) of variables and the corresponding subindex values (P_i) read from the rating curves at five different sampling stations (1, 4, 7, 8, and 10) are only presented in Table 1. These values were used in the computation of GRPI using different aggregation functions.

As all the rating curves were obtained from the expertise and judgment of the panelists, they are implicit nonlinear functions for which no direct mathematical function is available in the literature except for a few variables. Also, the nature and functional behavior of the subindex curves are not similar for all the selected variables. However, Swamee and Tyagi (2000) developed some mathematical relationships for uniformly decreasing, nonuniformly decreasing, and unimodal subindices based on the nature of subindex rating curves. But, such relationships have serious limitations due to uncertainties in accurate assessment of the various constants of the function. Cude (2001) also derived the subindex transformation formula for certain variables used in Oregon water quality index (OWQI) using nonlinear regression.

Aggregation Function

Aggregation has been defined as "the process of adding variables or units with similar properties to come up with a single number that represents the approximate overall value of its individual component" (Kumar and Alappat 2004). Aggregation functions can be of additive, multiplicative, minimum, or maximum operator forms (Ott 1978). As most of the water pollution indices reported in the literature is of the increasing scale form, they mostly use the additive form aggregation functions (Prati et al. 1971; Babcock and Nagda 1972; Inhaber 1974; Swamee and Tyagi 2000). Some of the water quality indices are of the decreasing scale form (Horton 1965; Brown et al. 1970; Dee et al. 1973; Walski and Parker 1974; Smith 1990). Like water quality indices, water pollution indices are independent of their functional forms and use all the three forms of aggregation functions (Ott 1978). Some investigators have used linear interpolation for BOD_5 and COD; nonlinear functions for suspended solids, NH_3, NO_3^-, and iron; and segmented nonlinear functions for other water quality variables (Ott 1978; Cude 2001).

A number of aggregation functions used by different investigators for the description of water quality or water pollution indices were collected from literature. A list of various aggregation functions along with their users and specific remarks are summarized in Table 2. Further detail on various aggregation functions of Table 2 can be seen in the research work of Ott (1978), Smith (1990), Swamee and Tyagi (2000), Cude (2001), Kumar and Alappat (2004), and Swamee and Tyagi (2007).

Criteria for Selection of Suitable Aggregation Function

The following aspects/criteria need to be considered in the selection of an appropriate aggregation function for the estimation of an aggregate index. These criteria are gleaned/judged from the literature (Ott 1978; Swamee and Tyagi 2000; Jollands et al. 2003; Kumar and Alappat 2004; Swamee and Tyagi 2007).

1. The most appropriate aggregation function is the one that is either free from or minimizes the overestimation (ambiguity), underestimation (eclipsing) and rigidity problems. Overestimation (ambiguity) problems arise when the aggregate index exceeds the critical level without any of the subindices exceeding the critical levels. Underestimation (eclipsing) problems exist when the aggregate index is too low and does not exceed the critical level. Rigidity problems arise due to inclusion of additional variables, and the aggregate index shows a low result because of the use of faulty aggregation function, indicating impaired water quality. In the present study, the rigidity problems are not given due consideration as the variables included are identified by expert opinion.

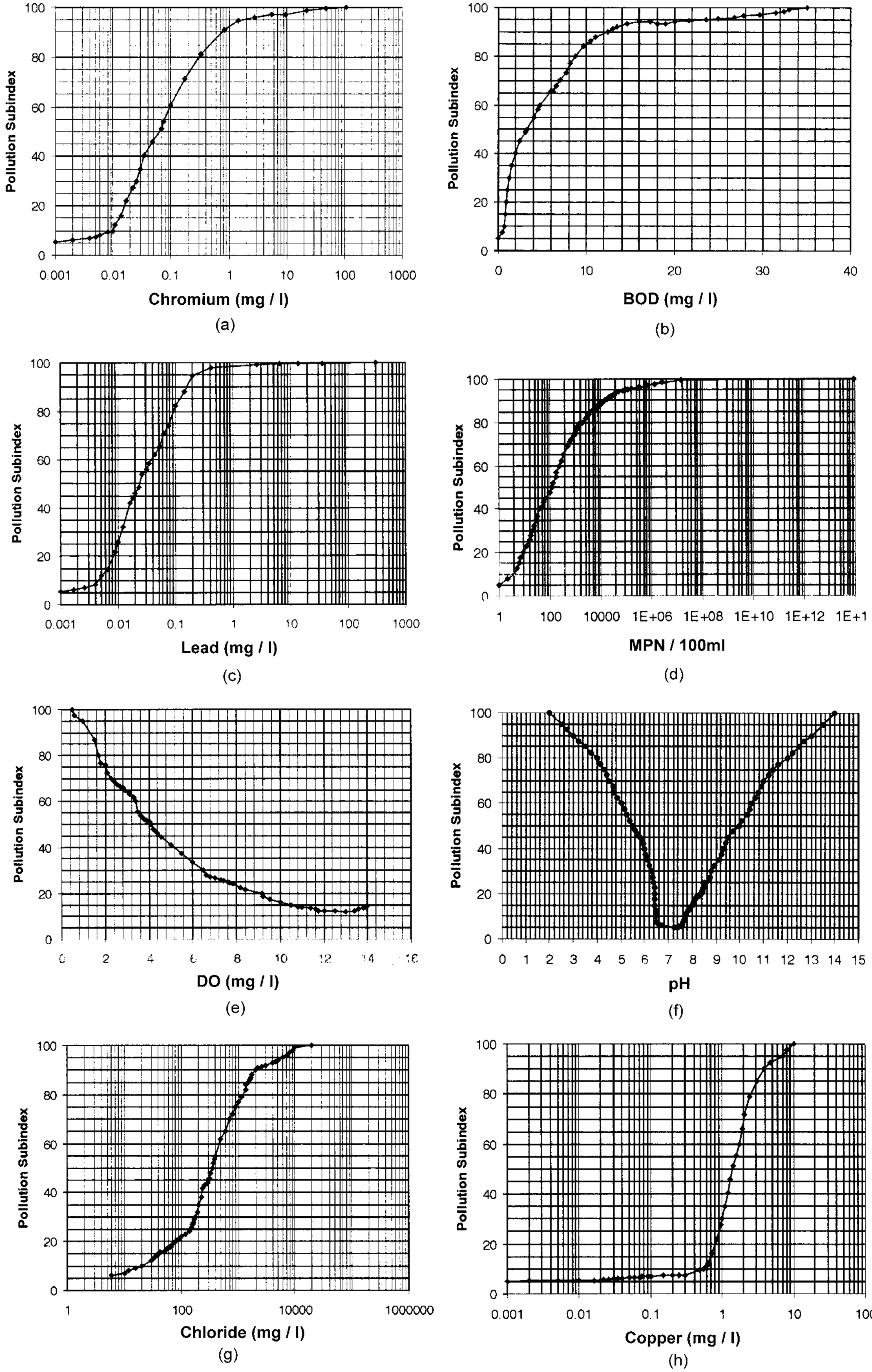


Fig. 2. Average subindex curves for river pollutant variables: (a) chromium; (b) biochemical oxygen demand (BOD_5); (c) lead; (d) MPN/100 mL; (e) dissolved oxygen (DO); (f) pH; (g) chloride; (h) copper; (i) total phosphorus (TPhos); (j) zinc; (k) total solids; (l) alkalinity; (m) hardness; (n) turbidity (NTU); (o) total Kjeldhal nitrogen (TKN); and (p) sulfate

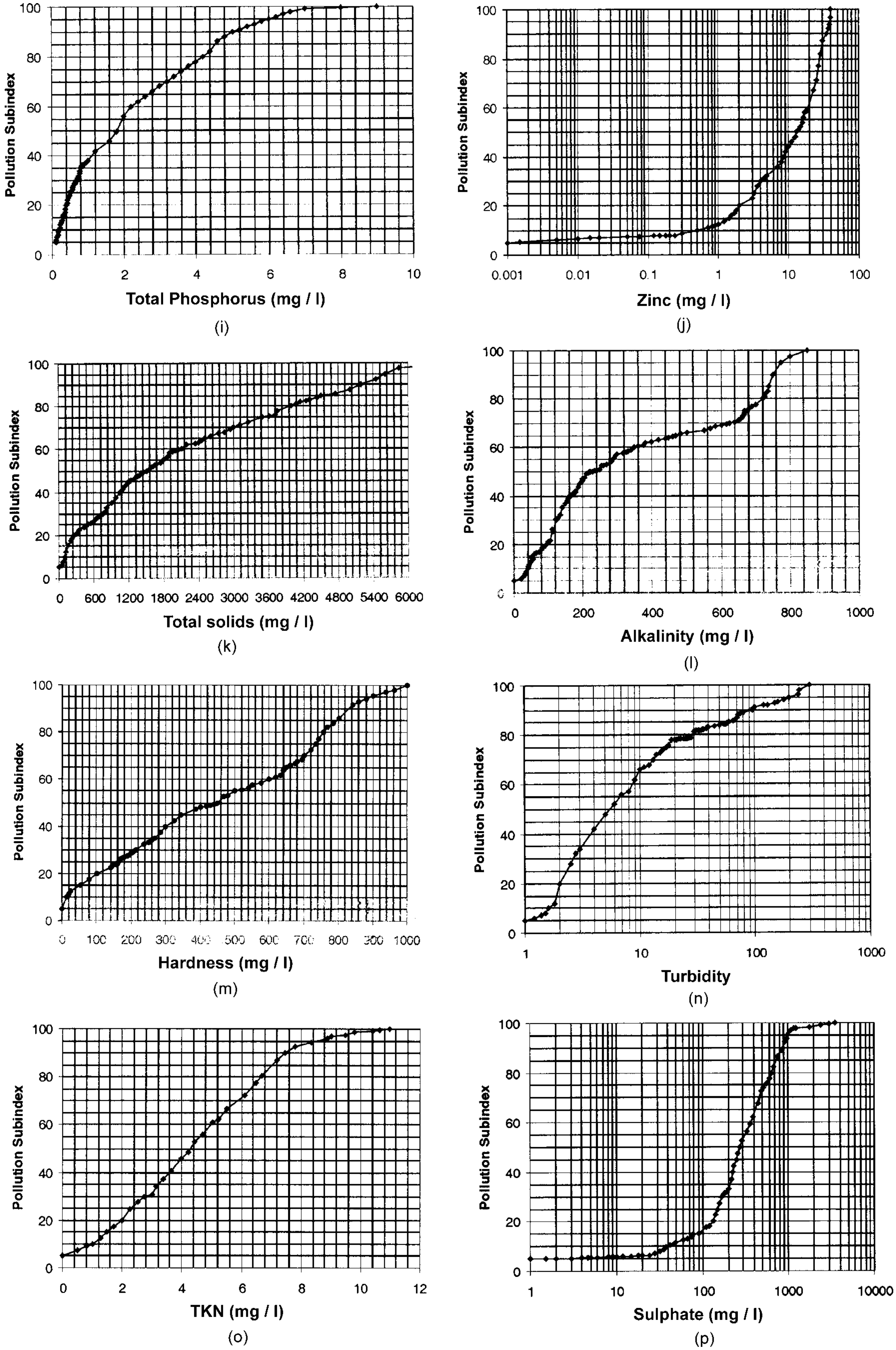


Fig. 2. (*Continued*).

Table 2. Aggregation Functions Used by Different Researchers for Water Quality and Pollution Indices

No.	Aggregation function	Function expression	Users	Specific remarks
1	Unweighted arithmetic mean function	$\mathrm{GRPI}_{\mathrm{uwa}}=\frac{1}{n}\Sigma_{i=1}^{n}P_i$	Brown et al. (1970); Prati et al. (1971); McDuffie and Haney (1973)	Ambiguous function; shows eclipsing region; simple but little flexibility; unsuitable for dichotomous subindices.
2	Weighted arithmetic mean function	$\mathrm{GRPI}_{\mathrm{wa}}=\Sigma_{i=1}^{n}W_iP_i$	Horton (1965); Brown et al. (1970); Prati et al. (1971); Dinius (1972); Dee et al. (1973); Inhaber (1974); Ott (1978); Ball and Church (1980); Egborge and Coker (1986); Mohan et al. (1996); Giljanovic (1999); Prasad and Bose (2001); Bardalo et al. (2001); Kumar and Alappat (2004)	Ambiguity free function; shows small eclipsing with large number of variables; not suitable for dichotomous subindices; widely used aggregation function.
3	Root sum power function	$\mathrm{GRPI}_{\mathrm{rsp}}=(\Sigma_{i=1}^{n}P_i^r)^{1/r}$	Swamee and Tyagi (1999); Kumar and Alappat (2004)	Shows reduced eclipsing but exhibit ambiguity problem; with increase in r, ambiguity decreases. If $r\rightarrow\infty$, it becomes ambiguity and eclipsity free function; use of aggregation function for $r>2$ is not practiced for aggregation of water pollution indices.
3a	Root sum power function ($r=2$)	$\mathrm{GRPI}_{\mathrm{r2sp}}=(\Sigma_{i=1}^{n}P_i^2)^{1/2}$		
3b	Root sum power function ($r=4$)	$\mathrm{GRPI}_{\mathrm{r4sp}}=(\Sigma_{i=1}^{n}P_i^4)^{1/4}$		
3c	Root sum power function ($r=10$)	$\mathrm{GRPI}_{\mathrm{r10sp}}=(\Sigma_{i=1}^{n}P_i^{10})^{1/10}$		
4	Weighted root sum power function	$\mathrm{GRPI}_{\mathrm{wrsp}}=(\Sigma_{i=1}^{n}W_iP_i^r)^{1/r}$	Kumar and Alappat (2004)	Exhibits slightly reduced ambiguity, unwidely used aggregation function.
4a	Weighted root sum power function ($r=4$)	$\mathrm{GRPI}_{\mathrm{wr4sp}}=(\Sigma_{i=1}^{n}W_iP_i^4)^{1/4}$		
4b	Weighted root sum power function ($r=10$)	$\mathrm{GRPI}_{\mathrm{wr10sp}}=(\Sigma_{i=1}^{n}W_iP_i^{10})^{1/10}$		
5	Root-mean-square function	$\mathrm{GRPI}_{\mathrm{rms}}=\left(\frac{1}{n}\Sigma_{i=1}^{n}P_i^2\right)^{1/2}$	Inhaber (1974); Kumar and Alappat (2004)	Exhibits small ambiguity problems.
6	Weighted root sum square function	$\mathrm{GRPI}_{\mathrm{wrss}}=\frac{(\Sigma_{i=1}^{n}W_iP_i^2)^{0.5}}{\Sigma_{i=1}^{n}W_i}$	Inhaber (1975); Kumar and Alappat (2004)	Exhibits small eclipsing problems.
7	Maximum operator function	$\mathrm{GRPI}_{\max}=\max[P_1,P_2,P_3-P_n]$	Smith (1990); Swamee and Tyagi (2000); Kumar and Alappat (2004)	No eclipsing problem but exhibit ambiguity for large number of variables; suitable for aggregation of air pollution subindices; limited application for water quality indices.
7a	Minimum operator function	$\mathrm{GRPI}_{\min}=\min[P_1,P_2,P_3-P_n]$	Smith (1990)	Free from eclipsity and ambiguity problems; performs similarly to maximum operator function; used in New Zealand as planting tool; suitable for decreasing scale index aggregation such as water quality indices.
8	Unweighted ambiguity and eclipsity free function ($r=0.4$)	$\mathrm{GRPI}_{\mathrm{uw0.4aef}}=(\Sigma_{i=1}^{n}P_i^{2.5})^{0.4}$	Swamee and Tyagi (1999); Kumar and Alappat (2004)	Eclipsity and ambiguity free function, limited application for air pollution indices; minimal ambiguity for $r=0.4$.
9	Weighted ambiguity and eclipsity free function $r=0.4$	$\mathrm{GRPI}_{\mathrm{w0.4aef}}=(\Sigma_{i=1}^{n}W_iP_i^{2.5})^{0.4}$	Kumar and Alappat (2004)	Eclipsity and ambiguity free function; limited application for leachate pollution indices.
10	Weighted average concentration function	$\mathrm{GRPI}_{\mathrm{wac}}=k\frac{\Sigma_{i=1}^{n}P_iC_i}{\Sigma_{i=1}^{n}C_i}$	Pesce and Wunderlin (2000); Debels et al. (2005)	Exhibits low sensitivity to changes in subindices for large number of variables; limited applications.
11	Subindex powered weight function	$\mathrm{GRPI}_{\mathrm{spw}}=\Sigma_{i=1}^{n}P_i^{W_i}$	Present study	Exhibit eclipsing problem.

Table 2. (*Continued.*)

No.	Aggregation function	Function expression	Users	Specific remarks
12	Unweighted multiplicative function	$GRPI_{uwm}=(\Pi_{i=1}^{n}P_i)^{1/n}$	Landwehr and Deininger (1976); Bhargava (1985); Kumar and Alappat (2004)	Exhibits small eclipsity problem, applied for comparison purposes only.
13	Weighted multiplicative or weighted geometric mean function	$GRPI_{wm}=\Pi_{i=1}^{n}P_i^{w_i}$	Walski and Parker (1974); Ball and Church (1980); Bhargava(1983, 1985); Dinius (1987); Swamee and Tyagi (2000); Kumar and Alappat (2004)	Nonlinear; ambiguity free but exhibits eclipsing at low weights and increasing scale indices; insensitive when applied to large number of variables.
14	Square root unweighted harmonic mean square function	$GRPI_{sruwh}=\sqrt{\dfrac{n}{\Sigma_{i=1}^{n}\dfrac{1}{P_i^2}}}$	Dojlido et al. (1994); Cude (2001)	Exhibits ambiguity problems; limited applications; a better function than geometric mean function.

2. The aggregation function selected for any environmental index shall also meet the following general criteria, i.e., it should (1) be sensitive to the changes in an individual variable throughout its range; (2) not be biased toward good or poor environmental quality; (3) consider weighting factors, as all variables included in the index are not equal contributors to water pollution; and (4) be relatively easy to use.
3. When competing aggregation functions produce similar results with respect to overestimation and underestimation, the most appropriate aggregation function will be the one that is mathematically simple (Jollands et al. 2003).
4. An aggregation approach is successful if all assumptions and sources of data are identified, the methodology is transparent and publicly reported, and an index can be readily disaggregated into the separate components with no information lost (Hammond and Adriaanse 1995).

To select the most appropriate aggregation function for the estimation of GRPI, the various available aggregation functions listed in Table 2 are applied to the observed variables on River Ganges for five selected stations (1, 4, 7, 8, and 10) only. The computed GRPI values are shown in Table 3. Keeping the above-mentioned considerations/criteria in mind, the various inappropriate functions were screened out. Similar results of computed GRPI values are also observed at another seven sampling stations located within the river stretch considered.

Results and Discussion

Preliminary Screening of Aggregation Functions

The most appropriate aggregation function for estimation of GRPI can be primarily analyzed and screened out on the basis of the following criteria.

Based on Ambiguity Criterion

From the GRPI values presented in Table 3, it is evident that root sum power ($GRPI_{r2sp}$), fourth root sum power ($GRPI_{r4sp}$), tenth root sum power ($GRPI_{r10sp}$), and unweighted ambiguity and eclipsity free ($GRPI_{uw0.4aef}$) aggregation functions produce ambiguous results indicating less polluted water conditions as more contaminated. Also, the computed GRPI values exceed the maximum reported individual pollutants' subindex values. Therefore, they are not considered to be the composite water quality indices. From Table 3, these GRPI values also exceed the theoretical range of GRPI, i.e., 5–100 except for the $GRPI_{r10sp}$ function. However, the $GRPI_{r10sp}$ function produces the least ambiguous result, followed by the functions $GRPI_{r4sp}$, $GRPI_{uw0.4aef}$, and $GRPI_{r2sp}$. Although, the $GRPI_{r10sp}$ function does not show much ambiguity in the results, but it cannot be used for calculating the GRPI value, as its results cannot be used to compare the fine gradations of river water pollutants.

In aggregation function $GRPI_{rsp}$, as the r value increases, the ambiguity (overestimation) becomes smaller (Ott 1978). For comparison, the $GRPI_{rsp}$ values for r values of 4 and 10 are also calculated. As these functions produce ambiguous results, it was considered worthwhile to use the weighted root sum power aggregation function ($GRPI_{wrsp}$) with r values of 4 and 10. The practical use of the root sum power aggregation function with r values more than 2 has, however, not been observed (Kumar and Alappat 2004). Swamee and Tyagi (1999) used $GRPI_{uw0.4aef}$ for aggregation of air pollution indices and suggested an r value of 0.4 for minimal ambiguity.

Table 3. Computed GRPI Values at Different Sampling Stations

Aggregation function	Station 1	Station 4	Station 7	Station 8	Station 10
$GRPI_{uwa}$	33.13	29.88	33.28	28.94	30.06
$GRPI_{wa}$	34.28	30.62	34.65	29.35	30.92
$GRPI_{r2sp}$	168.66	156.77	166.19	151.62	156.11
$GRPI_{r4sp}$	111.52	107.27	107.56	104.09	105.12
$GRPI_{r10sp}$	95.72	93.51	91.78	91.48	91.64
$GRPI_{wr4sp}$	56.00	54.30	54.24	51.84	53.32
$GRPI_{wr10sp}$	72.32	71.45	69.66	68.89	70.05
$GRPI_{rms}$	42.16	39.19	41.55	37.90	39.03
$GRPI_{wrss}$	42.97	39.94	42.57	38.05	40.21
$GRPI_{max}$	90.50	90.00	85.90	87.50	88.00
$GRPI_{min}$	7.00	7.50	6.50	6.00	7.50
$GRPI_{uw0.4aef}$	140.03	131.83	136.91	127.48	130.48
$GRPI_{w0.4aef}$	46.83	44.22	46.01	42.08	43.72
$GRPI_{wac}$	80.05	85.20	77.04	69.80	82.40
$GRPI_{spw}$	19.64	21.75	24.74	20.88	21.72
$GRPI_{uwm}$	24.24	22.41	26.19	21.48	22.53
$GRPI_{wm}$	25.55	19.48	19.68	19.42	19.49
$GRPI_{sruwh}$	13.99	13.71	14.24	12.47	13.31

Table 4. Preliminary Screening of Aggregation Functions at Different Sampling Stations

Criteria/screened functions	Station 1	Station 4	Station 7	Station 8	Station 10
Based on ambiguity criteria					
$GRPI_{r2sp}$	168.65	156.77	166.19	151.62	156.11
$GRPI_{uw0.4aef}$	140.03	131.83	136.91	127.48	130.48
$GRPI_{r4sp}$	111.51	107.27	107.56	104.09	105.12
$GRPI_{r10sp}$	95.72	93.51	91.78	91.48	91.64
Based on eclipsing criteria					
$GRPI_{spw}$	19.64	19.48	19.68	19.42	19.49
$GRPI_{sruwh}$	13.99	13.71	14.24	12.47	13.31
Based on constant functional behavior					
$GRPI_{min}$	7.00	7.50	6.50	6.00	7.00
$GRPI_{max}$	90.50	90.00	85.90	87.50	90.50
$GRPI_{wac}$	80.05	85.20	77.04	69.80	80.05
Based on non accountability of weight in functions					
$GRPI_{rms}$	42.16	39.19	41.55	37.9	39.03
$GRPI_{uwa}$	33.13	29.88	33.28	28.94	30.06
$GRPI_{uwm}$	24.24	21.75	24.74	20.88	21.72

On the basis of ambiguity criterion, the computed GRPI values using functions $GRPI_{r2sp}$, $GRPI_{r4sp}$, $GRPI_{r10sp}$, and $GRPI_{uw0.4aef}$ are found to be ambiguous and hence are short-listed. The computed GRPI values using these four aggregation functions at all the five sampling stations are summarized in Table 4.

Based on Eclipsing Criterion

The results of computed GRPI values using the square root unweighted harmonic mean square ($GRPI_{sruwh}$) and subindex powered weight ($GRPI_{spw}$) functions indicate high eclipsing of the river data and are presented in Table 4 for all the five sampling stations. From Table 4, it can be seen that the GRPI values are very low at all the sampling stations as compared to the other additive form aggregation functions. Also, it is gleaned from Table 3 that the results of the two multiplicative aggregation functions, i.e., unweighted ($GRPI_{uwm}$) and weighted ($GRPI_{wm}$), are relatively low and exhibit eclipsing problems in comparison to additive functions, such as unweighted ($GRPI_{uwa}$) and weighted ($GRPI_{wa}$) arithmetic mean functions. Although $GRPI_{uwa}$ and $GRPI_{wa}$ also suffer from the eclipsing problems, the eclipsity produced is relatively small as the number of the variables included in the aggregation function is large. Therefore, on the basis of the eclipsing criterion, the functions $GRPI_{sruwh}$ and $GRPI_{spw}$ could be easily ruled out for the estimation of GRPI.

Based on Constant Functional Behavior Criterion

The weighted average concentration ($GRPI_{wac}$), maximum operator ($GRPI_{max}$), and minimum operator ($GRPI_{min}$) aggregation functions show almost a constant functional behavior at a particular sampling station when the subindex values are varied from 5 to 100 for both chromium and sulfate (computations not shown). Thus, the estimation of GRPI using these aggregation functions appears to be insensitive to the changes in subindex values of the pollutant variables.

$GRPI_{max}$ takes the value of the largest of any of the subindices. Like $GRPI_{r10sp}$, $GRPI_{max}$ does not show much ambiguity of results, but it cannot be used to compare the microlevel concentrations of river water pollutants. Similarly, $GRPI_{min}$ is also an

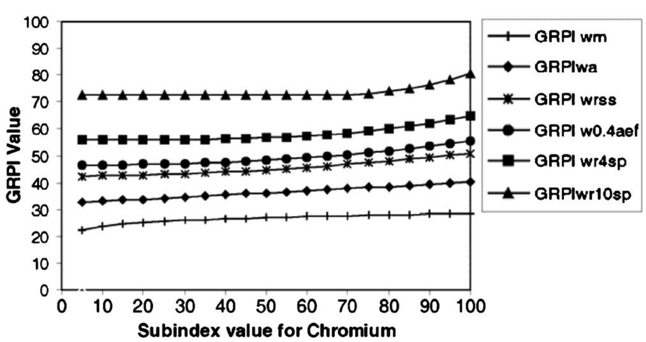


Fig. 3. Sensitivity of weighted multiplicative aggregation function, weighted arithmetic mean, weighted root sum square, weighted ambiguity and eclipsity free, weighted root sum power (r=4), and weighted root sum power (r=10) with respect to changes in subindex values of chromium

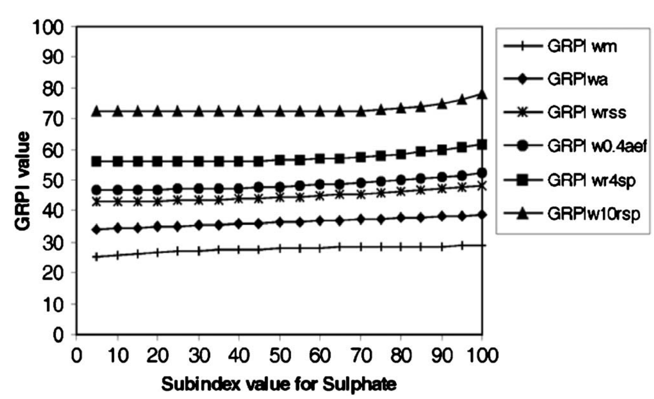


Fig. 4. Sensitivity of weighted multiplicative aggregation function, weighted arithmetic mean, weighted root sum square, weighted ambiguity and eclipsity free, weighted root sum power (r=4), and weighted root sum power (r=10) with respect to changes in subindex values of sulfate

Table 5. GRPI Values for Subindex Values of 5 and 100 at All Considered Sampling Stations for Chromium and Sulfate

Aggregation function	Station 1				Station 4				Station 7				Station 8				Station 10			
	Chromium		Sulfate		Chromium		Sulfate		Chromium		Sulfate		Chromium		Sulfate		Chromium		Sulfate	
	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$	$P=5$	$P=100$
$GRPI_{wm}$	22.500	28.508	25.068	28.858	21.212	26.876	21.917	25.230	22.459	28.456	25.619	29.492	20.332	25.761	21.293	24.513	21.328	27.023	22.103	25.445
$GRPI_{wa}$	32.701	40.206	34.163	38.628	30.221	37.726	30.475	34.940	32.283	39.788	34.512	38.977	28.952	36.457	29.300	33.765	30.529	38.034	30.807	35.272
$GRPI_{wrss}$	42.414	50.862	42.952	48.101	39.863	48.755	39.914	45.409	41.442	50.055	42.549	47.742	37.969	47.219	38.040	43.771	40.138	48.981	40.193	45.872
$GRPI_{w0.4aef}$	46.522	55.219	46.820	52.211	44.188	53.438	44.208	50.016	45.272	54.259	46.002	51.518	42.050	51.853	42.078	48.264	43.687	53.063	43.709	49.602
$GRPI_{wr4sp}$	55.959	64.868	56.003	61.747	54.300	63.824	54.301	60.496	54.050	63.671	54.236	60.450	51.837	62.358	51.839	58.760	53.323	63.230	53.324	60.029
$GRPI_{wr10sp}$	72.321	80.768	72.321	78.257	71.448	80.457	71.448	77.840	69.663	79.891	69.664	77.068	68.891	79.674	68.891	76.767	70.052	80.006	70.052	77.430

ambiguity and eclipsity free function (Smith 1990; Swamee and Tyagi 2000), but cannot be used to assess the sensitivity of function at the macrolevel concentrations of pollutant variables. Thus, based on the constant functional behavior criterion, the aggregation functions ($GRPI_{wac}$), ($GRPI_{max}$), and ($GRPI_{min}$) are not suitable for estimation of GRPI.

Based on Nonaccountability of Weights in Functions Criterion

A number of aggregation functions of Table 2 do not account for the weight of pollutant variables. As the weights are derived from the experts' opinions, it must be accounted for in the aggregation function for better composite water quality estimation. Besides, the previous nine prescreened aggregation functions, the functions $GRPI_{uwa}$, $GRPI_{rms}$, and $GRPI_{uwm}$ do not consider the weight of variables, and all the variables are assumed to be of weight unity, which appears to be unrealistic as different pollutant variables have different levels of significance from a water pollution point of view. The function $GRPI_{uwm}$ also suffers from the eclipsing problem. Thus, due to nonaccountability of pollutant's weight, the aggregation functions $GRPI_{uwa}$, $GRPI_{rms}$, and $GRPI_{uwm}$ are also ruled out for estimation of GRPI as shown in Table 4.

On the basis of the preliminary screening based on the ambiguity and eclipsing free criterion, constant functional behavior, and nonaccountability of the weight of variables, the use of 12 aggregation functions: $GRPI_{r2sp}$, $GRPI_{uw0.4aef}$, $GRPI_{r4sp}$, $GRPI_{r10sp}$, $GRPI_{max}$, $GRPI_{min}$, $GRPI_{sruwh}$, $GRPI_{spw}$, $GRPI_{rms}$, $GRPI_{wac}$, $GRPI_{uwa}$, and $GRPI_{uwm}$ could be ruled out for estimation of GRPI and their further inclusion in sensitivity analysis.

Therefore, the remaining six aggregation functions, namely the weighted arithmetic mean function ($GRPI_{wa}$), weighted root sum power function (r=4), $GRPI_{wr4sp}$, weighted root sum power function (r=10), $GRPI_{wr10sp}$, weighted ambiguity and eclipsity free function ($GRPI_{w0.4aef}$), weighted root sum square function ($GRPI_{wrss}$) and weighted multiplicative function ($GRPI_{wm}$) were subjected to sensitivity analysis to the changes in the individual pollutant's subindex value for the selection of the most suitable aggregation function for estimation of GRPI. All these functions account for the weight of pollutant variables and have relatively less ambiguity and eclipsing problems.

Sensitivity Analysis

The sensitivity analysis of the six selected aggregation functions—$GRPI_{wa}$, $GRPI_{wr4sp}$, $GRPI_{wr10sp}$, $GRPI_{w0.4aef}$, $GRPI_{wrss}$, and $GRPI_{wm}$—with respect to the changes in strength or concentration of two of the most and least significant pollutant variables (chromium and sulfate, respectively), is performed independently.

For performing the sensitivity analysis, the subindex value of chromium and sulfate is varied from 5 to 100 in the same data set for the average concentration of the variables (Table 1) for five selected sampling stations along the course of river and the GRPI values using above six aggregation functions are computed. The variation of GRPI values with respect to the changes in the subindex value of chromium and sulfate are plotted in Figs. 3 and 4, respectively, for Sampling Station 1 only. A similar plot could be observed (not shown here) for the remaining four sampling stations. The computed GRPI values for all the five stations (1, 4, 7, 8, and 10) at subindex values of 5 and 100 for both chromium and sulfates, respectively, are presented in Table 5. Using the GRPI values from Table 5, the percentage variation of the GRPI values

Table 6. Sensitivity Analysis Results of Selected Aggregation Functions for Change in Subindex Value of Chromium and Sulfate

	Percent change in GRPI Values for Chromium and Sulphate											
	Station 1		Station 4		Station 7		Station 8		Station 10		Average GRPI	
Aggregation function	Chromium	Sulfate	Chromium	Sulfate	Chromium	Sulfate	Chromium	Sulfate	Chromium	Sulfate	Chromium	Sulfate
$GRPI_{wm}$	26.70	15.12	26.70	15.12	26.70	15.12	26.70	15.12	26.70	15.12	26.70	15.12
$GRPI_{wa}$	22.95	13.07	24.83	14.65	23.25	12.94	25.92	15.24	24.58	14.49	24.31	14.08
$GRPI_{wrss}$	19.92	11.99	22.31	13.77	20.78	12.20	24.36	15.06	22.03	14.13	21.88	13.43
$GRPI_{w0.4aef}$	18.69	11.51	20.93	13.14	19.85	11.99	23.31	14.70	21.46	13.48	20.85	12.96
$GRPI_{wr4sp}$	15.92	10.26	17.54	11.41	17.80	11.46	20.30	13.35	18.58	12.57	18.03	11.81
$GRPI_{wr10sp}$	11.68	8.21	12.61	8.95	14.68	10.63	15.65	11.43	14.21	10.53	13.77	9.95

over the minimum value for the subindex variation ($P_i=5$) of chromium and sulfate are shown in Table 6 for all the five sampling stations along with their average values.

From Table 6, it can be seen that the aggregation function $GRPI_{wm}$=most sensitive one in comparison to other aggregation functions, showing an average change in GRPI values of 26.70 and 15.12%, respectively, for chromium and sulfate. The next most sensitive function is $GRPI_{wa}$, which shows an average of 24.31 and 14.08% variation in GRPI values for the two extreme pollutants, followed by functions $GRPI_{wrss}$, $GRPI_{w0.4aef}$, $GRPI_{wr4sp}$, and $GRPI_{wr10sp}$ in decreasing order of sensitivity.

From the plots in Figs. 3 and 4, it is observed that the function $GRPI_{wr10sp}$ is almost insensitive to the subindex variations for $P_i \leqslant 55$ and its sensitivity gradually improves for P_i above 60 for both chromium and sulfate. Therefore, this function does not give a better estimate of the overall water quality of River Ganges for changes in subindex values. Similar is the case with function $GRPI_{wr4sp}$, whose sensitivity is poor for $P_i \leqslant 35$, improves gradually up to $P_i \leqslant 55$ and improves rapidly for P_i above 60. These two functions do not reflect truly the fine gradations of water pollutant variables. The function $GRPI_{wrss}$ shows a nonlinear behavior to changes in subindex values and exhibits a low sensitivity at low subindex values ($P_i \leqslant 30$). The sensitivity improves nonuniformly at a faster rate beyond $P_i=70$. $GRPI_{w0.4aef}$ being a nonlinear function, has low sensitivity at low subindex values ($P_i \leqslant 25$) and its sensitivity increases rapidly beyond $P_i=40$. This function has not yet been used for aggregation of water quality or water pollution indices. The nonlinear function $GRPI_{wm}$, with highest sensitivity among six aggregation functions to percent changes in subindex variation (Table 6), is an ambiguity free function, but exhibit eclipsing problem at low weights. Also, the function's sensitivity drops down when the number of variables accounted for are large (Swamee and Tyagi 2000). From Figs. 3 and 4, it can be observed that $GRPI_{wm}$ has higher sensitivity at low subindex values ($P_i \leqslant 45$) and its sensitivity decreases rapidly for P_i above 50. Although the sensitivity analysis shows that the variation of $GRPI_{wm}$ values for the change in subindex values for chromium and sulfate is the highest, it suffers from the drawback that the function is nonlinear and shows biased results for higher subindex values. Therefore, these aggregation functions show inconsistent behavior and hence may not be useful for aggregating the water pollution indices especially when the fine gradations of water pollutants are essential.

The aggregation function, $GRPI_{wa}$ shows a uniform and true linear ($R^2=1$) behavior with variations in the subindex value for both chromium and sulfate, and ranks second in sensitivity among six considered aggregation functions (Table 6). Also, this function exhibits less eclipsing problem as compared to $GRPI_{wm}$. Thus, it can be inferred that the weighted arithmetic mean aggregation function ($GRPI_{wa}$)=most appropriate aggregation function for estimation of GRPI.

Conclusions

From the present study on selection of appropriate aggregation function for estimation of GRPI, it is inferred that the aggregation functions $GRPI_{r2sp}$, $GRPI_{r4sp}$, $GRPI_{r10sp}$, and $GRPI_{uw0.4aef}$ produce ambiguous results. The functions $GRPI_{sruwh}$ and $GRPI_{spw}$ exhibit high eclipsing problems. Aggregation functions $GRPI_{wac}$, $GRPI_{max}$, and $GRPI_{min}$ show almost constant functional variation at a particular sampling station irrespective of subindex variations. Also, the function $GRPI_{max}$ is ambiguity and eclipsity free, but it cannot be used for estimation of GRPI, as it is least sensitive to fine gradations of changes in concentrations of water pollutant variables. The functions $GRPI_{uwa}$, $GRPI_{rms}$, and $GRPI_{uwm}$ do not take into consideration the significance level or weight of all the variables, and assume that all the pollutant variables have the same significance level.

Based on the sensitivity analysis of six weighted aggregation functions, it is inferred that $GRPI_{wm}$ exhibit highest sensitivity followed by the function $GRPI_{wa}$. However, $GRPI_{wm}$ produces a nonlinear behavior and eclipsed/biased results. Other functions, $GRPI_{wrss}$, $GRPI_{w0.4aef}$, $GRPI_{wr4sp}$ and $GRPI_{wr10sp}$, are least sensitive in comparison to $GRPI_{wm}$ and $GRPI_{wa}$ to the changes in subindex values for both chromium and sulfate.

The aggregation function $GRPI_{wa}$ exhibits less eclipsing problems, a uniform and true linear behavior with variations in subindex values for both extreme pollutants (chromium and sulfate), and ranks second in sensitivity among six aggregation functions. Thus, the weighted arithmetic mean aggregation function ($GRPI_{wa}$) is found as the most suitable aggregation function for estimation of Ganga River pollution index.

Acknowledgments

This work was completed at and funded by Motilal Nehru National Institute of Technology (MNNIT), Deemed University, Allahabad, Uttar Pradesh, India. The writers are grateful to the Director, MNNIT for extending the required help and encouragements from time to time during this work. However, the writers are responsible for any errors, omissions, or interpretations in the manuscript.

Notation

The following symbols are used in this paper:

C = concentration or value of the pollutant variable;
n = number of variables;
P = subindex value;
R^2 = coefficient of determination; and
W = weight of pollutant variable.

Subscripts

i = ith parameter or variable;
max = maximum operator function;
min = minimum operator function;
rms = root-mean-square function;
rsp = root sum power function;
r2sp = root sum power function ($r=2$);
r4sp = root sum power function ($r=4$);
r10sp = root sum power function ($r=10$);
spw = subindex powered weight function;
sruwh = square root unweighted harmonic mean square function;
uwa = unweighted arithmetic mean;
uw0.4aef = unweighted ambiguity and eclipsity free function;
uwm = unweighted multiplicative or unweighted geometric mean function;
w0.4aef = weighted ambiguity and eclipsity free function;
wa = weighted arithmetic mean;
wac = weighted average concentration function;
wm = weighted multiplicative or weighted geometric mean function;
wrsp = weighted root sum power function;
wrss = weighted root sum square function;
wr4sp = weighted root sum power function ($r=4$); and
wr10sp = weighted root sum power function ($r=10$).

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