The Analytics of Changing Growth Rates

A vast amount of research has asked how and why the growth rates in the Indian economy have risen in recent decades. Implicit in much of that literature is the belief that if the growth rate has increased it must be because something underlying has changed – had some parameters embedded in the “structure” of the economy not changed, the growth rate would have been constant. This is, however, a presumption that may not be true. The search for structural “breaks” is the outcome of a preoccupation with steady states and constant rates of growth. To redress the balance this article provides some simple examples of models in which the rate of growth is never constant but changes endogenously over time. The lesson therefore is that changes in the growth rate have no necessary link with changes in the underlying economic regime or economic structure. This is not an India-specific point but is based on a general analytical argument.

1 Introduction

The most hackneyed remark that one can make about India’s growth rate is that it has been changing. The Indian economy has been growing at a faster pace in recent decades than it did earlier. Table 1 gives a snapshot of average growth rates since 1900.

Changing the length of the time interval over which the average is taken changes the value of the average. This would not have been the case if the underlying actual rate of growth per unit time had remained constant. It is clear that the actual rate of growth has been rising; the longer one looks back the lower the average.

These remarks are obvious truths that have little explanatory power. One would like to know what it is that has made improvements in the growth rate possible. There is a growing literature that discusses this question for post-Independence India. What is implicit in much of that literature is the belief that if the growth rate has increased it must be because something underlying has changed. In other words, had some parameters embedded in the “structure” of the economy not changed, the growth rate would have been constant. This is however a presumption that may not be true.

The search for “breaks” – e.g., breaking away from the “Hindu Rate of Growth” – is the outcome of preoccupation with steady states and constant rates of growth. To redress the balance we provide some simple examples of models in which the rate of growth is never constant but changes endogenously over time. The lesson of this note therefore is that changes in the growth rate have no necessary link with changes in the underlying economic regime or economic structure. This is not an India-specific point but is based on a general analytical argument.

That basic argument – which should be well known – is first set out verbally in the next section. The argument is then illustrated in Sections 3 and 4 below with concrete examples from economic theory, by analysing two elementary growth models.

2 Intrinsic Non-Constancy: The Basic Argument

Economic theory teaches us that typically the rate of growth is not constant over time. Excessive preoccupation with steady states and long-run equilibrium has blurred this lesson. On the steady state all economic variables grow at the same rate but outside the steady state different components of the economy will...
grow at different rates. It is this simple fact of growth heterogeneity that leads to an increase in the overall (i.e., the average overall components) rate of growth.1

The economy can be disaggregated in different ways depending on what we wish to study. For instance in a demand constrained set-up, GDP could be broken down into different demand components – consumption, investment, exports, government expenditures, etc. to see if growth is consumption-driven, investment-driven or export-driven.2 Similarly, the disaggregation that is relevant could be a sectoral one in terms of the “agriculture-industry-services” structure – e.g., if we wish to examine if growth deceleration is the result of a fall in the industrial rate of growth. Other examples can be given.

Whatever the nature of the disaggregation, the GDP growth rate is a weighted average of the growth rates of its different components, the weights being the respective shares of these components in GDP. For example, suppose all output is classified as “agricultural output” and “non-agricultural output” – for brevity, refer to the latter as “industrial output”. Then the rate of growth of GDP will be a weighted average of the agricultural growth rate and the industrial growth rate, with the weights being the respective shares in GDP of each output.

Assume that growth rates of both agriculture and industry are constant and given. If, for whatever reason, both outputs grow at the same rate then the shares of agricultural and industrial output in GDP will remain unchanged over time. Therefore in that case the GDP growth rate will indeed be constant.

But suppose instead that the two sectors are growing at different rates. For argument’s sake let agriculture be the faster growing sector. Then what is going to happen is this. Over time the share of agriculture in GDP will rise. That means the weight on the higher component (agricultural growth rate) will rise over time. Hence the weighted average (i.e., the GDP growth rate) will also rise over time. There has been no structural break, no shock, no change in external environment. Neither the industrial growth rate nor the agricultural growth rate has gone up; yet the GDP growth rate has.3

We now take up two simple models to see where the application of the basic argument takes us. These models are in fact embarrassingly simple so that the lessons to be learned can be stated in definitive terms. These lessons do not seem to have been applied in studying contemporary Indian growth. The first model (Section 3) is a demand-determined model with standard “multiplier” features. The second model (Section 4) is a supply-constrained model with standard “accelerator-multiplier” features.

3 Pure Demand Growth with Excess Capacity and Unemployment

We consider the simple Keynesian multiplier model in a closed economy in which there exists excess capacity and unemployment. The economy is therefore demand-constrained. We assume that investment is wholly autonomous, consumption has an autonomous component and the marginal propensity to consume is constant. If the consumption function is \( C = \bar{C} + \alpha Y \), then the equilibrium condition that demand = income gives

\[
D = Y = A/s \quad \text{...(i)}
\]

where \( A = I + \bar{C} \) represents autonomous demand. Taking this static model, we simply project it over time. Excess capacity and unemployment are assumed to persist over time. The proportional rates of growth autonomous consumption \((\dot{g}_c)\) and autonomous investment \((\dot{g}_i)\) are both given exogenously: \( \dot{g}_c = \alpha \) and \( \dot{g}_i = \beta \).

Let \( \mu = \frac{I}{I + \bar{C}} \) be the share of investment expenditure in total autonomous demand. For positive \( I \) and \( \bar{C} \), we have \( 0 < \mu < 1 \).

Writing \( g_t \) for the rate of growth of \( x \), the relation \( A = I + \bar{C} \) implies

\[
g_A = \mu \beta + (1-\mu)\alpha
\]

From the multiplier relation (1) we have \( g_g = g_A - g_c \). Assume \( g_c = 0 \) and let \( g = g_A = g_c \). Then \( g = \mu \beta + (1-\mu)\alpha \).

Note that \( \mu \) is a function of time such that

\[
\frac{d\mu}{dt} = \mu (1 - \mu) (\beta - \alpha) = \mu (1 - \mu) \theta, \quad \text{where} \quad \theta = \beta - \alpha \quad \text{...(3)}
\]

Since \( 0 < \mu < 1 \) \( \Rightarrow \mu (1 - \mu) > 0 \), it follows from (3) that for \( 0 < \mu < 1 \),

(i) \( \theta > 0 \Rightarrow \mu > 0 \) and \( \mu \to 1 \) as \( t \to \infty \).

(ii) \( \theta < 0 \Rightarrow \mu < 0 \) and \( \mu \to 0 \) as \( t \to \infty \).

If \( \mu = 0 \) or 1, then \( \dot{\mu} = 0 \).

These properties are displayed in Figure 1.

**Figure 1: Share of Investment in Autonomous Demand**

From (2) we have \( \dot{g} = \beta - \alpha \dot{\mu} = \theta \dot{\mu} = \mu (1 - \mu) \theta^2 \quad \text{...(4)} \)

It follows from (2) and (4) that for \( 0 < \mu < 1 \),

(i) \( \alpha \neq \beta \), i.e., \( \theta \neq 0 \Rightarrow \dot{g}(t) > 0 \);

(ii) \( \dot{g}(t) \geq 0 \) for all \( t \geq 0 \) since \( \theta \) and \( \dot{\mu} \) always have the same sign.

Therefore the growth rate is non-decreasing, regardless of the values of \( \alpha \) and \( \beta \).

3.1 Properties of Demand-Determined Growth

Let \( \bar{g} = \max \{\alpha, \beta\} \) be called the “leading rate”. Let \( g_{\min} = \min \{\alpha, \beta\} \) be called the “laggard rate”.

We take note of the following properties of the growth path. Property 1: The GDP growth rate increases over time. If \( 0 < g(0) < 1 \), then \( g_{\min} < g(t) < \bar{g} \) and \( \dot{g}(t) > 0 \) for all \( t \geq 0 \).

The result is intuitive. By virtue of the simple multiplier relation, the GDP growth rate \( g \) is equal to the rate of growth of autonomous demand \( A \). The latter is a weighted average of its two components \( \bar{C} \) and \( I \), the weights being the respective component shares. If \( \bar{C} \) and \( I \) are positive, then \( 1 > g(0) > 0 \) and that implies \( g(t) > g_{\min} \). The weight on the faster growing component keeps increasing over time, pulling up the average rate of growth towards it.

Note: It is possible that the growth rate is negative; that requires \( g_{\min} \) to be negative.
Properties II: The GDP growth rate converges to the leading rate.

(a) If \( g(0) > g_{\min} \), then \( g(t) \to \frac{g}{g_{\min}} \) as \( t \to \infty \).

(b) If \( g(0) = g_{\min} \), then \( g(t) = g_{\min} \) for all \( t \geq 0 \).

These properties are illustrated through Figures 3(a) and 2(b). The growth rate in a demand-driven framework is pulled up (or may be termed as “driven”) by whichever is the faster growing component of autonomous demand, i.e., by the leading rate. Provided that the economy starts strictly above \( g_{\min} \), the growth rate \( g(t) \) converges to \( g_{\min} \). Else it is stuck at \( g_{\min} \).

Property III: (a) A one-shot jump in \( g_{\min} \) at \( t = 0 \) shifts \( g(0) \) and \( g(t) \) up in the short run (i.e., for all \( t < \infty \)).

(b) The long-run growth rate is unaffected; \( \frac{g}{g_{\min}} \) is unaffected and the shifted path still converges to \( g_{\min} \).

These properties are illustrated through Figures 3(a), 3(b) and 4.

3.2 Structural Breaks?

Properties I and II above clearly bring out the fact that the overall growth rate – which is the average of its two components, consumption and investment rate of growth – is never constant. Save in the long run, the rate of growth is never almost never) constant. If the initial growth rate is not the leading or laggard rate, then the growth rate will rise over time to the level of the leading rate. The increase in the growth rate is clearly not the result of changes in any exogenous parameter or economic environment or structural equation of the model. The increase in growth rate is entirely endogenous to the model.

The issue of “structural breaks” in growth is about disruptions to a pre-existing pattern of observations. Having perceived a disruption on the observational plane one then examines if that disruption can be linked to some known change(s) of importance – such as a major change in economic policy (e.g., liberalisation) or in some crucial parameter (e.g., a technological breakthrough such as the Green Revolution). In terms of the model studied above, it is the solution path \( Y(t) \) that constitutes the observed GDP growth path. In this model, the level and the growth rate of GDP are both continuous functions of time. Moreover, the solutions \( Y(t), g(t), \frac{g}{g_{\min}}(t) \) depend in a continuous manner on the exogenously specified time paths \( C(t), I(t) \) and \( s(t) \). A “break” is a jump or discontinuity in the solution path. That could happen if one of the exogenously specified time paths were to change in a discrete manner. Roughly speaking, a discrete change is a very large change in a very short interval of time.

Sometimes, the growth series is arbitrarily broken up into discrete intervals and an average growth rate calculated for each interval. This amounts to approximating a non-linear graph by a series of piece-wise linear graphs. The approximation of course leads to successive jumps in the growth rate, but these do not reflect structural breaks (Figure 5, p 67).

4 Full-Capacity Growth: Harrod-Domar Models

An obvious question in the context of the above demand-determined growth model is this: For how long is it possible to have positive
net investment in the face of persistent excess capacity? This is obviously relevant when the investment growth rate is the higher of the two. In case the consumption growth rate is higher, one is led to ask how it is possible to maintain high rates of consumption growth without expanding productive capacities at sufficiently high rates over time. There must be some production-demand consistency that puts an upper bound on the consumption growth rate.

Let us drop the assumption of everlasting excess capacities. A simple exercise is to move to the class of models that is best described as the “Harrod-Domar models” (see Domar 1946; Harrod 1939; and also Ray 2007, ch 3). The production-demand consistency question in these models is easily explained. Any positive net investment adds to capacity. In order for these new capacities to be utilised fully, there must be an appropriate increase in aggregate demand. That requires an appropriate increase in investment demand – recall the multiplier relation. A constant level of net investment will cause capacity to grow at an arithmetic rate, but leave demand unchanged. Hence a constant path of investment tends to generate excess capacity. Excess capacity would induce a cut back in the level of investment, making the problem worse. A constant level of investment is simply not sustainable over time; investment has to grow at a sufficiently rapid rate for investment to be sustainable. The Harrod-Domar model of short run growth was designed to address this issue and it was shown that investment is required to increase at a sufficient rate for investment to be sustainable. The Harrod-Domar model of short run growth was designed to address this issue and it was shown that investment is required to increase at a sufficient rate for investment to be sustainable. The Harrod-Domar model of short run growth was designed to address this issue and it was shown that investment is required to increase at a sufficient rate for investment to be sustainable. The

4.1 A Generalised Harrod-Domar Model with Autonomous Consumption

We consider a generalised Harrod-Domar model by incorporating autonomous consumption expenditure. Assume that in a closed economy the productivity of capital $B$, the marginal propensity to save $s$ has exogenously given constant paths over time. Let $Y = BK$ be the production relation where $(Y/B)$ is the capital-output ratio. With full-capacity utilisation, market clearing entails $sBK = K + \bar{C}$. Divide by $K$ and let $g_K = I/K$. Then $g_K = sB - (\bar{C}/K)$ ... (5)

From (5), $I = sBK - \bar{C}$, hence $\dot{I} = sB \dot{K}$, i.e., $gI = sB$.

Hence full capacity equilibrium requires investment to grow at a constant rate determined by the saving ratio and productivity of capital. This rate is exactly the Harrod-Domar rate.

Let $sB \equiv \beta$. In fact $gI = \beta$ is the original Harrod-Domar rate of growth (in the absence of any autonomous consumption). Next introduce growth of autonomous consumption. Let the growth rate of autonomous consumption $\bar{C}$ be denoted by $\alpha$ as before.

Differentiating (5), with fixed $\beta$, we get

$$\dot{g}_K = \frac{\dot{\bar{C}}}{\bar{K}} + \frac{\bar{C}}{\bar{K}^2} \dot{K} = \frac{\dot{\bar{C}}}{\bar{K}} + \left(\frac{\bar{C}}{\bar{K}} \right) \frac{\dot{\bar{K}}}{\bar{K}} = -\alpha \frac{\bar{C}}{\bar{K}} + \frac{\bar{C}}{\bar{K}} g_K$$

$$g_K = \frac{\bar{C}}{\bar{K}} (g_K - \alpha) = (\beta - g_K) (g_K - \alpha) \quad \ldots (6)$$

From (6), $g_K$ is positive either when $\alpha < g_K < \beta$ or $g_K < \alpha$.

However note from (5) that it is always the case that $g_K < \beta$ for positive $\bar{C}$. Hence for the growth rate to be increasing we must have $\alpha < g_K < \beta$.

4.2 Properties of Growth in the Generalised Harrod-Domar Model

Property iv

The GDP growth rate increases over time, i.e., $\dot{g}(t) > 0$ provided that $\beta > \alpha$.

Property v

(a) If $g(0) > \alpha$ and $g(0) \neq \beta$ then $\dot{g}(t) > 0$ for all $t \geq 0$ and $\lim_{t \to \infty} g(t) = \beta$. The long-run rate of growth is the Harrod-Domar rate.

(b) If $g(0) < \alpha$, then $g(t) \to -\infty$ as $t \to \infty$. If $g(0) = \alpha$, then $g(t) = \alpha$ for all $t \geq 0$. If $g(0) = \beta$ then $g(t) = \beta$ for all $t \geq 0$.

Properties iv and v are displayed graphically in Figure 6.

Property vi (see Figure 7).
4.3 Discussion: Consumption-driven Growth?

In this model, full-capacity growth requires $\beta > \alpha$. One question that has been asked – especially in the “informal” literature – is whether contemporary Indian growth has been spurred by investment or by consumption. In the Harrod-Domar model, growth has got to be “investment-driven”. The long-run rate of growth is the investment growth rate, an increase in the investment growth rate has the effect of pulling the growth path up, and the effect of an increase in the consumption growth rate is exactly the opposite, i.e., the growth path is pulled down.

As with the previous model of pure demand growth, in this model too the growth rate is almost never constant. The growth rate will rise on its own – except when it is too low initially – simply because it is lower than the Harrod-Domar full-capacity level. In a sense the growth rate rises because initially autonomous consumption is too high relative to the capital stock – see (5) above.

Consider an alternative path with a higher rate of consumption growth $\alpha'$. Then $g(t)$ is lower on the alternative path for all $t > 0$. This means that $g(t)$ is lower for all $t > 0$ as well. However, the long-run rate of growth remains unaffected.

This result is at variance with that in the model of pure demand-determined growth. In the demand model a rise in $\alpha$ does cause the short-run growth rate to go up though it does not affect the long-run rate of growth (assuming $\beta > \alpha$). With excess capacity, a rise in any component of autonomous demand is helpful. However, in capacity constrained models such as the Harrod-Domar model a rise in consumption demand must reduce investment since output is given. Hence the growth rate must go down in the short run.

Therefore growth can be consumption-driven only under conditions of excess capacity.

5 Conclusions

The theoretical results obtained here can be read in conjunction with the large and growing literature on Indian growth. The Indian economy has been growing at a faster pace in recent decades than it did in the first few decades after Independence. Two discernible phases of economic growth in India since Independence have been recognised: 1950 to 1980 and 1980 to 2008. Compared to the preceding 30 years, there was a distinct step-up in rates of growth for GDP and GDP per capita. The earlier period has been commonly described as “the Hindu growth rate era” in which the average growth rate was 3.5%. During the period from 1980 to 2008 the average growth rate has gone up to 6%. Econometric exercises, such as those of Wallack (2003); Hausmann, Pritchett and Rodrik (2004); Rodrik and Subramanian (2004) and Virmani (2004), suggest that the structural break in the trend rate of growth takes place around the early 1980s.

Models developed in this paper throw up growth rates that change endogenously over time. A change in the observed average rate of exponential growth does not necessarily signal a “structural break”. In the pure demand-determined model the GDP growth rate increases whenever the consumption and investment growth rates differ from each other. The faster growing component of demand pulls up the overall GDP growth rate (which is an average of the component growth rates) towards it. Similarly in the generalised Harrod-Domar model, increase in the growth rate of GDP is ensured because investment growth rate must exceed the consumption growth rate. In both cases, the increase in growth rate is endogenous.

These models can also be used to provide tight definitions of consumption-driven growth and investment-driven growth, and to examine the possibility of characterising growth in these terms. Our finding is that growth can be consumption-driven only under conditions of excess capacity.